Accuracy Assessment of the Position Tracking Filter

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BIOGRAPHY

Mohamed Khalaf-Allah received the M.Sc. degree in computer engineering in September 2004 and the Ph.D. degree (Dr.-Ing.) in communications engineering in October 2008 both from the Leibniz University of Hannover, Germany. Since August 2007, Dr. Khalaf-Allah is employed as a research scientist at the Dept. of Aerospace Engineering, Institute of Flight Guidance, TU Braunschweig, Germany. The research interests of Dr. Khalaf-Allah include Positioning & navigation technologies, sensor & data fusion, filtering techniques & estimation theory, autonomous mobile robots, intelligent transportation systems and sensor network applications.

1. INTRODUCTION

In the nineties, the GPS were made accessible for commercial applications. Moreover, the EU is most likely to follow the US and Japan in requiring high positioning accuracy of mobile emergency calls when the Galileo system will be fully operational. However, the benefits of global navigation satellite systems (GNSS) could be limited where position information is still needed due to obscured view to satellites, lack of a GNSS receiver in the mobile terminal (MT) to be located or degraded accuracy caused by multipath. It is believed that signals of opportunity (SOP), e.g. GSM positioning and dead-reckoning (DR) navigation, are the answer to the missing GNSS location updates and a lot of research is being carried out in this field. In case of GNSS outage, GSM signals and DR data are advantageous in terms of availability and coverage. Furthermore, DR is a desirable option for security and commercial applications that value position information.

In [1] a position tracking algorithm, based on the recursive Bayesian filter, was proposed to maintain mobile location information during GPS signal blocking. A backup to GPS during signal outage with comparable accuracy could be achieved using fusion of inertial measurement unit (IMU) raw data with already existing cell-ID based methods and map-matching. The initial position was assumed as the last available GPS position fix. The main task of the algorithm was to compensate for IMU or DR data errors using map-matching. The position tracking filter (PTF) is a mobile-based solution, where map information is provided by network operators. However, the technique could be run as network-based if the IMU data is uploaded from the MT to the operator network.

The feasibility of the PTF was investigated in [1] by fusing simulated IMU data, real-world cell-ID information from cellular wireless networks and map-matching. Radio maps generated by radio propagation prediction tools were used off-line to determine road areas covered by every cell antenna in the test area. Experiments determined the range of acceptable IMU data errors that would allow reliable positioning when using real IMU data.

In this contribution, we will introduce an assessment tool, which is capable of computing the theoretical achievable positioning accuracy of estimators that involve dynamics measurements, such as the PTF under hand. This assessment tool is an extension of the well known Cramér-Rao lower bound (CRLB) that accounts for available dynamics measurement data, and is called the posterior Cramér-Rao lower bound (PCRLB). More information about the PTF, assumed world and motion models are given in [1].

The PCRLB along with the definition of its terms are given in section 2. Section 3 presents the numerical results of the PCRLB computations. Finally, we conclude in section 4.

2. THE POSTERIOR CRAMÉR-RAO LOWER BOUND

The PTF is a nonlinear recursive filter. In order to asses its performance using the Cramér-Rao bound (CRB) approach, we have to compute the Fisher information matrix (FIM) also recursively, and the resulting lower bound is called the posterior Cramér-Rao lower bound (PCRLB) [2], [3], [4].

Unlike the CRLB for the deterministic (non-random or constant) parameters, the estimator to be assessed is not required to be unbiased. The only requirement is that both sides of the Cramér-Rao inequality must exist. Also it is assumed that the state transition probability density function (pdf) exists and is twice differentiable w.r.t. its arguments. Similarly, it is supposed that the measurement pdf exists and is twice differentiable w.r.t. the state at the desired time index.

The PCRLB is the inverse of the recursively computed FIM. The recursive FIM computation is given as

$$\begin{aligned} FIM_{t+1} &= (Q + E[F_t \cdot FIM_t^{-1} \cdot F_t^T])^{-1} \\ &+ E[H_{t+1}^T \cdot R^{-1} \cdot H_{t+1}] \end{aligned} . \qquad (1)$$

Where F_t is the Jacobian matrix of the state transition function $f(s_t)$ evaluated at the true values of the state s_t, H_{t+1} is the Jacobian matrix of the measurement function $h(s_{t+1})$ evaluated also at the true values of the state s_{t+1}, Q is the covariance matrix of the process noise, R is the covariance matrix of the measurement error and $E[\cdot]$ denotes the expectation.

The state transition matrix of the PTF at time t is defined as

$$f(s_t) = \begin{bmatrix} x_t + d_t \cdot \cos\phi_t \\ y_t + d_t \cdot \sin\phi_t \\ d_t \\ \phi_t \end{bmatrix} . \qquad (2)$$

Where x_t, y_t, d_t, and ϕ_t are the MT's x, y coordinates, travelled distance, and orientation respectively, that build the state at time t. Therefore, F_t is written as

$$F_t = \frac{\partial f(s_t)}{\partial s_t} = \begin{bmatrix} 1 & 0 & \cos\phi_t & -d_t \cdot \sin\phi_t \\ 0 & 1 & \sin\phi_t & d_t \cdot \cos\phi_t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \qquad (3)$$

The measurement function $h(s_{t+1})$ contains only the timing advance (TA) measurement from the main or serving base station (BS), which roughly estimates the distance between the MT and the serving BS. The definition of $h(s_{t+1})$ is given as

$$\begin{aligned} h(s_{t+1}) &= \left\lfloor \sqrt{(x_{t+1} - x_{t+1}^{BS})^2 + (y_{t+1} - y_{t+1}^{BS})^2} \right\rfloor \\ &= \left[D_{t+1}^{TA} \right] \end{aligned} . \qquad (4)$$

Where x_{t+1}, y_{t+1}, x_{t+1}^{BS}, y_{t+1}^{BS}, and D_{t+1}^{TA} are respectively the MT's x, y coordinates, serving BS's x, y coordinates, and the TA distance measurement of the serving BS all at time $t+1$. Therefore, H_{t+1} is written as

$$\begin{aligned} H_{t+1} &= \frac{\partial h(s_{t+1})}{\partial s_{t+1}} \\ &= \begin{bmatrix} \frac{x_{t+1} - x_{t+1}^{BS}}{D_{t+1}^{TA}} & \frac{y_{t+1} - y_{t+1}^{BS}}{D_{t+1}^{TA}} & 0 & 0 \end{bmatrix} \end{aligned} . \qquad (5)$$

The covariance matrix of the process error Q is given as

$$Q = \begin{bmatrix} \sigma_x^2 & 0 & 0 & 0 \\ 0 & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_d^2 & 0 \\ 0 & 0 & 0 & \sigma_\phi^2 \end{bmatrix} . \qquad (6)$$

Where σ_x^2, σ_y^2, σ_d^2, and σ_ϕ^2 are the error variances of the MT's x, y coordinates, and total translation, and orientation respectively. Note that $\sigma_d^2 = \sigma_x^2 + \sigma_y^2$.

The measurement noise R is calculated as

$$R = \left[\sigma_{D^{TA}}^2 \right] . \qquad (7)$$

Where $\sigma_{D^{TA}} \approx 277$ m assuming a TA measurement error of $\frac{1}{2}$ bit [5].

3. EXPERIMENTAL RESULTS

As indicated in [1], measurements have been carried out in a GSM 1800 MHz network by a pedestrian along a route of about 1940 m long. Received signal strength (RSS) measurements of the serving BSs and up to six neighboring stations along with GPS position fixes for ground truth have been logged into a file for later offline simulations. The GPS fixes have been used to simulate DR measurements in order to investigate the suitability of IMU employment.

We have investigated the performance of the PTF by varying the standard deviation of translation measurement error σ_{trans} from 1% to 10% of the performed translation and the standard deviation of orientation measurement error σ_{orient} between 1° and 6°. All experiments have been repeated 100 times in order to get reasonable results. The mean positioning error as suggested by the PCRLB, illustrated in Figure 1, is always less than 10 m for all values of σ_{trans} and σ_{orient} up to 4°. For values of σ_{orient} greater than 4°, the PCRLB is always less than 12 m. The mean positioning error of the PTF is between 15m and 20m, is always in the vicinity of the PCRLB and is never more than 10 m away from the bound. The nearly similar performance in all cases is due to the 5 m resolution of the utilized radio profile maps, which smoothes the positioning errors within the ranges assumed for σ_{trans} and σ_{orient}.

4. CONCLUSIONS

We derived and computed the posterior Cramér-Rao lower bound (PCRLB) in order to investigate the theoretical achievable accuracy of a positioning filter, called the position tracking filter (PTF), which involves dynamics measurements. While both the Cramér-Rao lower bound (CRLB) and dilution of precision (DOP) calculations are commonly used to predict reachable accuracies of static positioning, the PCRLB provides a mathematical tool to recursively calculate the Fisher information matrix (FIM) and thus is able to assess the performance of dynamic positioning, i.e. positioning systems that utilizes measurements from, e.g. an inertial measurement unit (IMU). Our position tracking algorithm is tested outdoors within the communication cellular system GSM and utilizes cell-ID, timing advance (TA), radio profile maps and simulated dead-reckoning (DR) measurements. Positioning accuracies (mean absolute error) are comparable to the GPS performance, and are in the vicinity of the theoretical achievable accuracy as predicted by the PCRLB.

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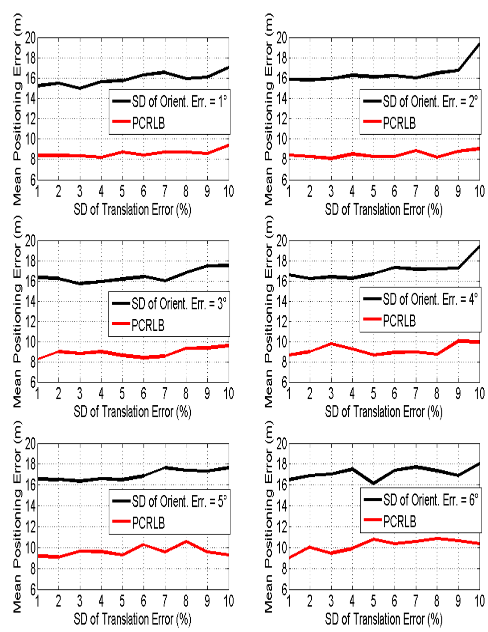

Fig. 1: Mean positioning errors of the PCRLB and the PTF.