IMU low cost calibration method

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BIOGRAPHY

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ABSTRACT

The aim of this work is to propose a low cost method to calibrate a low cost IMU (Inertial Measurement Unit). Nowadays, the use of Micro Electro-Mechanical System instruments is widely accepted, owing to their great flexibility linked to their cheap costs. On the other hand, such peculiarities imply a loss of accuracy and performances. In order to have an instrument being completely low cost, also a low cost calibration procedure is required.
In the present paper experiments on low cost IMU are discussed; the tested instrument is composed of a cluster of almost orthogonal accelerometers and gyroscopes, based on MEMS technology. For both the accelerometers and the gyros, the proposed calibration method is based on the use of magnitude of reference quantities instead of the single component value.

INTRODUCTION

Calibration is the procedure for comparing instrument output with known reference information about the quantity to measure; it is a fundamental operation for its strong influence on the results of a measurement. For this reason all builders carry out appropriate calibration procedures to produce a precise calibration certificate. On periodical basis, the operation of calibration has to be repeated, either by the same builder or by users employing self-made algorithms. The calibration process often needs expensive tools. Currently, an interesting research topic is the development of algorithms and methodologies to perform the initial procedures according to the "low cost philosophy".

For an accelerometer, the reference information is usually the apparent gravity vector, sum of gravitational and centrifugal accelerations, known for a fixed place on Earth. A common calibration method is performed at a known fixed site on Earth and it consists in setting the accelerometer cluster on several different and precisely known attitudes (other methods can perform calibration during navigation and by means of Kalman filtering). A set of measures is carried out by each accelerometer of the cluster and the average is calculated for every orientation. Three equations per orientation are attained, in which the calibration parameters are the unknowns. The cluster orientation and the number of positions are chosen to obtain the required independence and redundancy of the equations set with respect to unknowns.

The calibration parameters result from the model used to represent the relationship between measure and reference information, which is linked to the instrument accuracy. For a MEMS low cost accelerometer, the scale factor non-linearity can be considered negligible, and the relationship between measure and reference depends on 9 parameters: 3 misalignment angles, 3 scale-factors, 3 biases. To use the aforesaid method, the knowledge of the local gravity vector components for every orientation is necessary. Therefore, the precise knowledge of inclination angles is required. If a precisely adjustable 6 degree of freedom platform is unavailable, the calibration can be performed using the gravity magnitude, accurately known at fixed site, as reference. In this case, from every roughly known orientation, only one equation is achieved. This method can be considered a “low cost” calibration procedure, because there is no need to use precise attitude platform or other instruments, except for the inertial sensor and a common PC. The described procedure is particularly suitable for low cost sensors calibration. Generally, 18 or more orientations are performed, in order to obtain a system of redundant equations, solved using least squares method. Every equation is previously linearized around initial values of the unknowns, estimated from theoretic parameters from the instrumental data sheet, such as the bit number in output from the A/D converter and the full-scale.

For the gyro cluster calibration, the same procedure is implemented; the only notable difference is that the known constant rotation has to be imposed to calibrate the gyros. The proposed low cost solution uses the gyro cluster placed on a record player in several roughly known attitudes.

A comparison between the low cost and the precise attitude (from instrumental calibration certificate) methods is carried out to confirm the validity of the proposed procedure; data set of different accuracy are used to define the algorithm flexibility.

PROCEDURES & ALGORITHMS

The experimentation discussed was conducted by means of a MTi-Motion Tracker sensor from XSens Technologies. The instrument makes use of MEMS technology and consists of 3 accelerometers, 3 gyros and 3 magnetometers. The device is equipped with a factory software which permits an user-friendly communication between the sensor itself and a processor, and a simple data management. Such a software was used during the tests in order to acquire raw data from the sensor.

Before the delivery to the user, the factory usually conducts the calibration process employing an appropriate calibration table and providing the calibration certificate. The aim of the research is to validate the proposed low-cost calibration procedure comparing the obtained results with the data set from the factory certificate.

The first step of a calibration process consists in the identification of the suitable model describing the relationship between measure and reference information. The selected model is strictly linked to the instrument accuracy.

For a MEMS low cost sensor (such as the employed XSens MTi) the scale factor non-linearity can be considered negligible and so the relationship measure – reference can be modelled as:

$$\bar{s} = T \cdot K \cdot \left(\bar{s}_{out} - \bar{b} \right) \quad (1)$$

where:

$\bar{s}$ is the actual gravity vector (known)

$\bar{s}_{out}$ is the measured accelerations vector (known)

T is the alignment matrix, transforming the non-orthogonal accelerometer axes in the orthogonal platform frame

K is the scale-factor matrix

$\bar{b}$ is the bias vector

The foresaid model is valid both for the accelerometers and the gyroscopes.

In detail the calibration model for the accelerometer cluster is:

$$\begin{pmatrix} s_X \\ s_Y \\ s_Z \end{pmatrix} = \begin{pmatrix} 1 & \alpha_Z & \alpha_Y \\ 0 & 1 & \alpha_X \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} k_X & 0 & 0 \\ 0 & k_Y & 0 \\ 0 & 0 & k_Z \end{pmatrix} \cdot \left[\begin{pmatrix} s_{outX} \\ s_{outY} \\ s_{outZ} \end{pmatrix} - \begin{pmatrix} b_X \\ b_Y \\ b_Z \end{pmatrix} \right] \quad (2)$$

The unknowns are 9 calibration parameters, that, introduced in (2), force the measured accelerations to agree with the reference quantity, that is the local gravity value. The unknowns are collected in the vector X:

$$\overline{X} = [\alpha_X \quad \alpha_Y \quad \alpha_Z \quad b_X \quad b_Y \quad b_Z \quad k_X \quad k_Y \quad k_Z]^T \tag{3}$$

The non-linear equation system (2) can be solved by linearization around an estimation of the unknown vector (3). Then, system (2) becomes:

$$A_{[3\times9]} \cdot d\overline{X}_{[9\times1]} \approx -\bar{s}_{[3\times1]} + \overline{N}_{[3\times1]} \tag{4}$$

$$\overline{N} = \begin{bmatrix} k_{XS}(s_{outX} - b_{XS}) + \alpha_{ZS}k_{YS}(s_{outY} - b_{YS}) + \alpha_{YS}k_{ZS}(s_{outZ} - b_{ZS}) \\ k_{YS}(s_{outY} - b_{YS}) + \alpha_{XS}k_{ZS}(s_{outZ} - b_{ZS}) \\ k_{ZS}(s_{outZ} - b_{ZS}) \end{bmatrix} \tag{5}$$

$$A^T = \begin{bmatrix} 0 & -k_{ZS}(s_{outZ} - b_{ZS}) & 0 \\ -k_{ZS}(s_{outZ} - b_{ZS}) & 0 & 0 \\ -k_{YS}(s_{outY} - b_{YS}) & 0 & 0 \\ -k_{XS} & 0 & 0 \\ \alpha_{ZS}k_{YS} & k_{YS} & 0 \\ \alpha_{YS}k_{ZS} & \alpha_{XS}k_{ZS} & k_{ZS} \\ -(s_{outX} - b_{XS}) & 0 & 0 \\ -\alpha_{ZS}(s_{outY} - b_{YS}) & -(s_{outY} - b_{YS}) & 0 \\ -\alpha_{YS}(s_{outZ} - b_{ZS}) & -\alpha_{XS}(s_{outZ} - b_{ZS}) & -(s_{outZ} - b_{ZS}) \end{bmatrix} \tag{6}$$

where vector N and matrix A depend on the measured acceleration vector and on the initial calibration vector.

Equations (4) could be solved using least squares method, if we precisely know the components of local gravity vector for each orientation. The precise knowledge of inclination angles is necessary. The calibration vector is obtained by:

$$\overline{X} = \overline{X}_S + d\overline{X} \tag{7}$$

If a precisely adjustable 6 degree of freedom platform is not available, the calibration can be performed by using the gravity magnitude, accurately known at fixed site. For this purpose, equation (4) has to be further modified to get free from the gravity components. From 3 scalar equations (4), depending on gravity components, a scalar equation, only depending on gravity magnitude, can be derived.

From equation (4):

$$s^2 = \bar{s}^T \cdot \bar{s} = \overline{N}^T \cdot \overline{N} - \overline{N}^T \cdot Ad\overline{X} - d\overline{X}^T A^T \cdot \overline{N} + d\overline{X}^T A^T \cdot Ad\overline{X} \tag{8}$$

A first order approximation of equation (8) allows to neglect the last term on the right side, being $\overline{N}^T \cdot Ad\overline{X}$ and $d\overline{X}^T A^T \cdot \overline{N}$ scalar terms:

$$s^2 \approx \overline{N}^T \cdot \overline{N} - 2\overline{N}^T \cdot Ad\overline{X} \tag{9}$$

$$(\overline{N}^T A)_{[1x9]} \cdot d\overline{X}_{[9x1]} \approx \frac{1}{2}(\overline{N}^T \cdot \overline{N} - s^2)_{[1x1]} \tag{10}$$

By the use of at least 9 scalar equations, such as (10), the unknown correction vector can be computed and placed in (7) to achieve the calibration parameters.

By this method, each cluster orientation generates an equation as (10). Several different attitudes have to be carried out, being not necessary to know the orientation angles accurately.

Generally, 18 or more orientations are performed, in order to obtain a system of redundant equations (10), solved by least squares method: each raw vector with 9 components (N^TA) is a raw of the design matrix H, and each scalar $0.5(N^TN\text{-}s^2)$ is a component of the known terms vector (formula (11)).

$$H_{[18\times9]} \cdot d\overline{X}_{[9\times1]} = l_{[18\times1]} \tag{11}$$

The algorithm is summarized in figure 1.

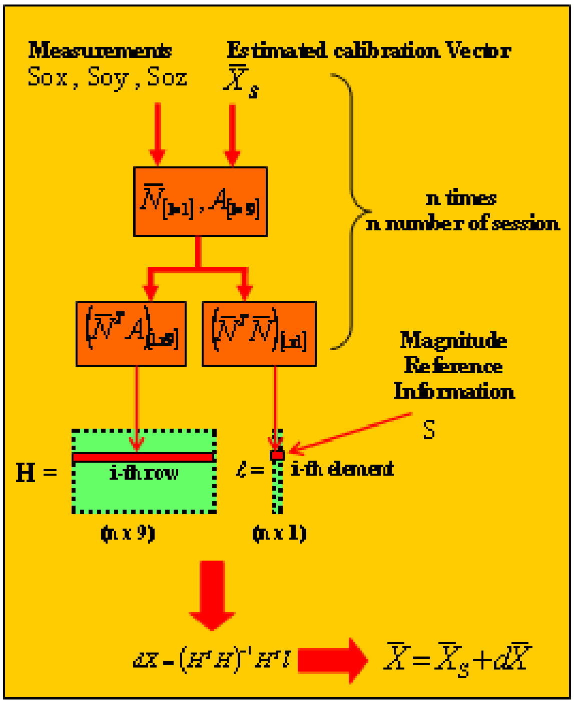

Fig. 1 – Algorithm Scheme

The theoretic parameters derived from instrument data sheet can be used as initial estimated calibration vector. They allow the conversion from raw data to suitable unit of measurement (usually m/s^2).

The misalignment angles are very small, so a first estimate value is zero. A first estimate of the scale factors is the conversion factor, defined as the ratio between full-scale in m/s^2 and the biggest output value from the instrument A/D converter. A first bias estimate is the "false origin", introduced because the output range in m/s^2 is symmetric as regards the origin, while the A/D converter output range is not.

So if the accelerometer full-scale (from datasheet) is [-y, +y] m/s^2 and the device is equipped with a n-bit A/D converter, producing one of the integers $\{0,1,\ldots,2^n-1\}$ for each sampled input, the estimated calibration vector is:

$$\bar{X}_S = [0 \;\; 0 \;\; 0 \;\; (2^n-1)/2 \;\; (2^n-1)/2 \;\; (2^n-1)/2 \;\; 2y/(2^n-1) \;\; 2y/(2^n-1) \;\; 2y/(2^n-1)]^T \qquad (12)$$

If another initial estimate is used, the algorithm can not converge to the right solution or can even diverge.

For the XSens MTi used, the accelerometers full-scale is [-50, +50] m/s^2, the gyros full-scale is [-150, +150] deg/s, the bit number of the A/D converter is 16.

To find the final solution several iterations are usually necessary, where the solution at $(i-1)^{th}$ step is used as initial estimate at i^{th} step. The iterative process is stopped when relative difference between the residual quadratic forms at subsequent steps is smaller than a fixed tolerance. The residual vector and the residual quadratic form q are defined as:

$$\bar{v} = H \cdot d\bar{X} - \bar{l} \quad ; \quad q = \bar{v}^T \cdot \bar{v}$$

If the process converges, residual quadratic form decreases with iterations. The criterion for stopping iterations can be $\dfrac{q_{i-1} - q_i}{q_1} \leq 0.0005$.

NON-ORTHOGONAL TO ORTHOGONAL REFERENCE SYSTEMS TRANSFORMATION

Physically, each accelerometer has a sensitive axes. Generally, in an Inertial Measurement Unit a cluster composed of three accelerometers is included. The input accelerometer axes (x_a, y_a, z_a) are mounted nearly orthogonal. The cluster frame has to be made orthogonal, obtaining the so-called platform system (x_p, y_p, z_p).

In general the transformation from a non-orthogonal reference system to a generic orthogonal one needs six rotational angles (the transformation between 2 orthogonal frames needs 3 rotation angles only).

$$\begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} = \begin{pmatrix} \hat{x}_p \cdot \hat{x}_a & \hat{x}_p \cdot \hat{y}_a & \hat{x}_p \cdot \hat{z}_a \\ \hat{y}_p \cdot \hat{x}_a & \hat{y}_p \cdot \hat{y}_a & \hat{y}_p \cdot \hat{z}_a \\ \hat{z}_p \cdot \hat{x}_a & \hat{z}_p \cdot \hat{y}_a & \hat{z}_p \cdot \hat{z}_a \end{pmatrix} \cdot \begin{pmatrix} x_a \\ y_a \\ z_a \end{pmatrix} \qquad (13)$$

Equation (13) expresses the general rotation between two frames, where the elements of the rotational matrix are the direction cosines.

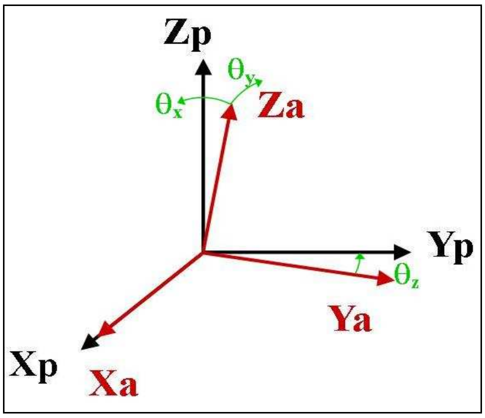

Fig. 2 – Non-Orthogonal to Orthogonal Frame Transformation

In order to make the frame orthogonal, three rotations are sufficient. Let x_a coincide with x_p and y_a lie in the $x_p y_p$ plane, as figure 2 displays; in this case only three rotations are necessary and the direction cosines matrix becomes:

$$\begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} = \begin{pmatrix} 1 & \sin\theta_z & \cos\theta_x \sin\theta_y \\ 0 & \cos\theta_z & \cos\theta_y \sin\theta_x \\ 0 & 0 & \cos\theta_x \cos\theta_y \end{pmatrix} \cdot \begin{pmatrix} x_a \\ y_a \\ z_a \end{pmatrix} \qquad (14)$$

The misalignment angles can be considered small; therefore, a linearization is applied:

$$\begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} = \begin{pmatrix} 1 & \theta_z & \theta_y \\ 0 & 1 & \theta_x \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_a \\ y_a \\ z_a \end{pmatrix} \qquad (15)$$

Equation (15) explains the expression of the alignment matrix T in (2).

As the implemented calibration method employs only the magnitudes of the reference information (gravity vector and imposed angular velocity), it is not possible to trace back the accelerometer and the gyro frames to a common orthogonal platform frame. The accelerometers and the gyros reference systems have to be made orthogonal separately, using equation (15). This produces two distinct frames: an orthogonal accelerometer system and an orthogonal gyro one.

RESULTS

To test the developed algorithm on accelerometers, 2 sets of input data have been carried out with roughly known attitudes. The first set is achieved with an orientation accuracy of about 1 degree (using a woody wedge to obtain a coarse 45° angle, showed in figure 3), and the second with an orientation accuracy of about 10° (without any supports).

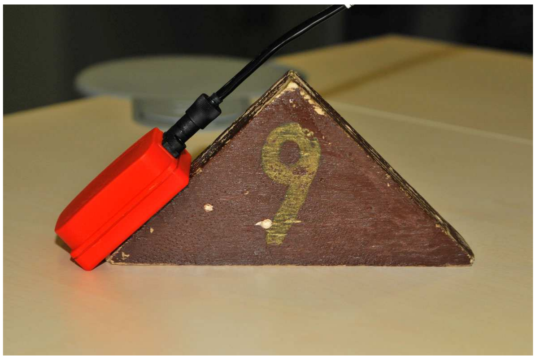

Fig. 3 – Woody Wedge for Coarse 45° Attitudes

The results of the calibration procedure, using the different measurements, are shown in table 1.

1TH MEASUREMENT SET								
α [‘]			b [bit]			k^{-1} [(m/s^2)/bit]		
-0.0135	-0.0036	0.0005	33610	33382	32063	547.6	552.8	550.6
σ_α = ±1’			σ_b = ±0.6			σ_k = ±2.6e-7		
2TH MEASUREMENT SET								
α [‘]			b [bit]			k^{-1} [(m/s^2)/bit]		
-0.0139	-0.0038	0.0003	33608	33380	32060	548.6	553.0	550.5
σ_α = ±2.5’			σ_b = ±1.3			σ_k = ±5e-7		
CALIBRATION CERTIFICATE								
α [‘]			b [bit]			k^{-1} [(m/s^2)/bit]		
—	—	—	33652	33395	32034	549.7	552.7	550.4

Tab. 1 – Accelerometers Calibration Results

The results obtained with measurement sets characterized by different accuracy are very similar, showing the algorithm flexibility: the procedure is valid even with weak data. The more accurate set is characterized by variances with almost one order of magnitude lower than the other.

It is noteworthy that the computed calibration parameters are comparable with the calibration certificate values, provided by the builder and obtained by means of a precisely adjustable 6 DOF platform.

Only few (4-5) iterations are necessary to find the final solution (fig. 4). If the used initial estimate is not equal to (12), the number of iterations raises (fig. 5) or the algorithm can not converge to the right solution or can even diverge (fig. 6). Several trials have been carried out to identify a convergence interval for the 3 parameter types: misalignment angles, biases and scale factors. The intervals for the 3 parameters are shown in table 2. It is noteworthy that all the intervals are not symmetric around the data sheet initial estimates (α_0, $Bias_0$, k_0). Choosing the initial estimate vector outside the convergence intervals, the calibration parameters do not converge to the right solution or diverge (fig. 6).

Convergence Intervals
Misalignment Angles [α_0-18°, α_0+19°]
Biases [$Bias_0$-1500, $Bias_0$+2500]
Scale Factors [k_0-0.0006, k_0+0.03]

Tab. 2 – Convergence Intervals

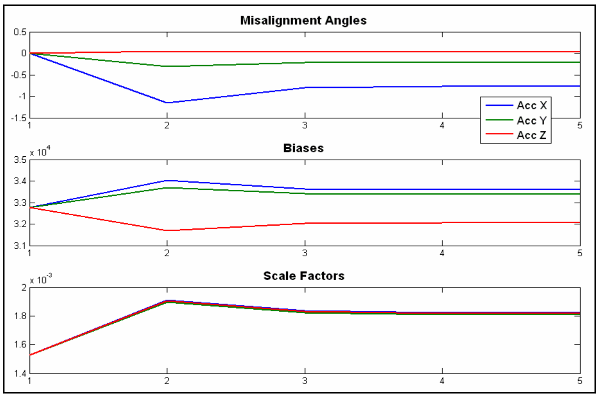

Fig. 4 – Iterations with Initial Estimate derived from Instrument Data Sheet

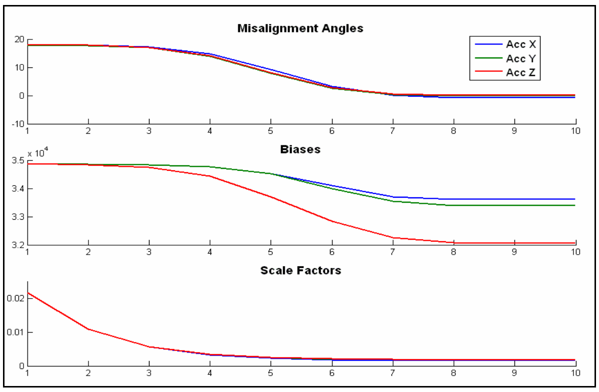

Fig. 5 – Iterations with Initial Estimate inside the Convergence Interval

To test the “low cost” calibration procedure on gyros, a constant angular velocity was imposed by placing the sensor on a 16 rpm record player at 18 different positions (fig. 7).

The realized angular velocity was measured using a video recorded with a webcam.

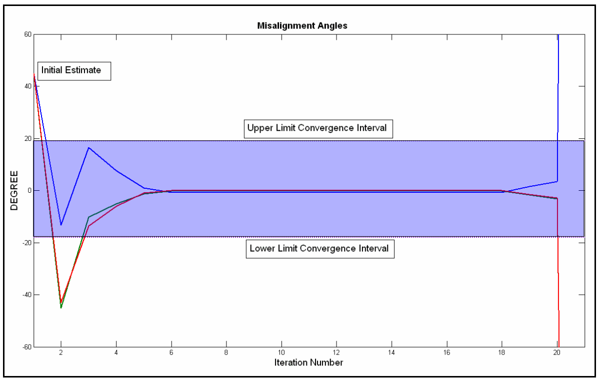

Fig. 6 – Iterations with Initial Estimate outside the Convergence Interval for Misalignment Angles

Fig. 7 – A 16 rpm Record Player to Impose a Constant Angular Velocity

A mark was drawn on the rotating plate and another on the fixed part of the record player. The angular velocity was evaluated by counting the number of frames between successive alignments of the marks. The fps was known, that is 15 fps. Because of the method inaccuracy, standard deviations of the calibration parameters are significantly worsened for the gyros. The results of the gyros calibration procedure are shown in table 3.

Gyro MEASUREMENT SET								
α [‘]			b [bit]			k^{-1} [(rad/s)/bit]		
-0.0089	-0.0026	0.0130	28585	36273	34528	10835	9592	9575
$\sigma_\alpha = \pm 0.5°$			$\sigma_b = \pm 60$			$\sigma_k = \pm 4.5e\text{-}7$		
CALIBRATION CERTIFICATE								
α [‘]			b [bit]			k^{-1} [(rad/s)/bit]		
—	—	—	28575	36338	34681	9651	9402	9417

Tab. 3 – Gyros Calibration Results

CONCLUSIONS

A low cost algorithm for the calibration of a low cost IMU has been tested. The procedure employs only the magnitudes of the necessary reference information, avoiding the use a 6 DOF rate table.
To validate the method, the calibration parameters are compared with calibration certificate parameters provided by the factory of the employed instrument.

For the MEMS accelerometers, the low cost method guarantees satisfactory results. The obtained parameters are very near to the calibration certificate, and the algorithm is very flexible, working even with week data.
Accuracy obtained in the MEMS gyros calibration is not very satisfactory, owing to the coarse angular velocity impressed and to the rough model employed.

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