Heuristic Search Techniques to Improve Multi-Constellation RAIM Performances

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BIOGRAPHY

Roberto Ronchini holds a degree in Computer Science Engineering and a Ph.D. in Systems Engineering, both from Sapienza - University of Rome. He joined Telespazio in 2001 as Navigation Engineer and has been working since then in the fields of GNSS integrity (RAIM and EGNOS), and, more in general, navigation-related algorithms design and implementation.

INTRODUCTION

RAIM (Receiver Autonomous Integrity Monitoring) is an integrity validation concept, whose aim is to provide the GNSS user with immediate and autonomous integrity information. It is based on processing standard measurement data (navigation observables) collected by a receiver and that will be further used for positioning.

The idea behind RAIM techniques is to use available redundant measurements to check the consistency of an over-determined positioning solution.

In particular, Fault Detection and Exclusion (FDE) algorithms attempt to find a set of redundant measurements that doesn't cause failure detection.

Traditional algorithms, such as the baseline Van Graas' RAIM algorithm, work searching the best among subsets of 6 satellites. This philosophy has been then extended to larger subsets or an "all-in-view" operation to cope with multi-constellation scenarios.

The "all-in-view" concept assumes using all satellites for positioning when there's no detected failure, and falling back to the best subset without failure if a failure into the full set is detected.

In case of multi-constellation single-failure mode, a maximum of n subsets could need to be explored when adopting the all-in-view approach.

When operating in multi-failure mode, the number of subsets may drastically increase, ranging from few thousand to hundred millions, depending on the subset size and the number of satellites in view.

In other words, the algorithm implies the solution of a hard combinatorial optimization problem, where the dimension of search space is strictly related to the number "n" of simultaneously collected measurements.

When such a number increases, which is the case of a combined constellation scenario, traditional techniques may result ineffective due to the largeness of the search space, yielding to problems with high complexity in terms of calculation time, exponentially increasing with "n" and thus computationally intractable.

In a frame where exhaustive search techniques may result unfeasible and a simple "random search" would perform poorly, heuristic optimization approaches can provide sub optimal solutions in a reasonable time and with moderate computational effort, reducing receiver processing workload.

Simulated annealing and genetic algorithms represent two of the algorithms which were most experienced and studied during last years, in particular when dealing with scheduling optimisation problems.

The aim of this paper is to show how such heuristic optimization approaches may be used to increase search performances and drastically reduce computational complexity of multi-constellation RAIM algorithms, providing sub-optimal solutions in a reasonable time and with moderate calculation effort.

In this work, a traditional RAIM FDE algorithm has been modified by replacing the classical extensive search with a new search engine based on the simulated annealing concept.

This technique has been preferred to other ones (e.g. genetic algorithms) because it proved to be very simple and effective when applied to low and medium complexity search problems.

Raw data have been coherently reproduced and several different combinations of biased measurements have been used to simulate and predict algorithm behaviour.

INTRODUCTIVE CONCEPTS ABOUT RAIM FDE METHODS

It is important to remember that the proposed approach may be directly applied to all the RAIM algorithms which perform some kind of search among subsets of satellites.

In this work the Van Graas' RTCA-approved FDE algorithm, as derived in [9], has been chosen as test-bed for algorithm simulation as it is very popular among the user community as well as still widely used in the practice.

This paragraph will briefly introduce some basic concepts about Van Graas' algorithm, referring the reader to specific literature to have more details about RAIM techniques and their main properties ([2], [3] and [8]).

In the following, the reader is assumed having sufficient familiarity with RAIM terminology and, more in general, with basic satellite navigation integrity concepts.

Fault Detection and Exclusion, or *FDE*, as its name suggests, protects against position errors by trying to find a set of redundant measurements that doesn't cause detection, and excluding the failed satellite(s) from the positioning calculations. FDE approach doesn't require identification and isolation of the malfunctioning satellite.

The baseline algorithm is based on the parity space concept. It can be summarized as follows:

1. In normal operation, with $n \geq 6$ satellites in view, the system may compute a navigation solution from the subgroup of six satellites having the smallest protection level R_p. If fewer than 6 satellites are in view, or if no subgroup can be found with a protection level less than the admissibility threshold, a navigation failure is declared.
2. The test statistic, formed by the greatest of successive projections of a parity vector onto the characteristic bias lines of the satellites being used in the navigation solution, is calculated.
3. If the test statistic P_m is greater than the detection threshold, an alarm is triggered. The algorithm searches for a subset of 6 satellites that will satisfy the above condition. The search is made in ascending order of the protection levels of all possible subsets, and stops only when a satisfactory subset is discovered. This subset is then used for both navigation and integrity monitoring until another alarm is triggered.
4. If no satisfactory subset can be found, an integrity failure is declared.

In the algorithm above, a subset is considered admissible if its protection level is less than the alarm limit for the considered system.

Protection level and test statistic are classical RAIM concepts. The Protection level is defined as the position error that will not be exceeded for the fault detection function, in accordance with the missed and false alert requirements. The test statistic is a parameter which expresses a measure of inconsistency between pseudoranges as observed by the user.

In the following it will be proposed a novel way to enhance the search logic of this particular FDE RAIM algorithm in order to cope with the multi-constellation case, hence dealing with a larger number of satellites, avoiding performance degradation and computational complexity increase. Nevertheless, this approach may be also applied to any other RAIM algorithm based on combinatorial search.

THE APPLICATION OF HEURISTIC OPTIMIZATION TECHNIQUES

When dealing with complex search problems, simple linear operation-research algorithms or traditional search techniques may result ineffective; this is the case of NP-hard class problems.

These aspects have motivated researchers in extensively using heuristic approaches, in particular nature-based ones.

Heuristic optimization approaches may provide sub-optimal solution in a reasonable time and with moderate computational effort, reducing computer's workload.

Presumably, simulated annealing and genetic algorithms represent two of the algorithms most experienced and studied during last years (in particular when dealing with scheduling optimization problems) and many other different nature-based approaches have been developed in pursuit of improve their performances [1].

Both simulated annealing and genetic algorithms are nature-based stochastic computational techniques; simulated annealing is based on thermodynamics, genetic algorithms are based on natural evolution modelling.

The major advantages of these algorithms are their broad applicability, flexibility, ease of implementation, and the potential of finding near-optimal solutions.

In this work, simulated annealing has been preferred to genetic algorithms because it is one of the simplest optimisation techniques and at same time it proved to be very efficient in solving problem of such complexity.

Simulated annealing

As its name implies, the Simulated Annealing exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure (the annealing process) and the search for a minimum in a more general system.

Simulated annealing is a well-known heuristic technique for efficiently solving optimization problems, where simulation techniques from statistical physics are used to solve combinatorial optimization problems [5].

The algorithm is based upon that of Metropolis [6], which was originally proposed as a means of finding the equilibrium configuration of a collection of atoms at a given temperature.

Simulated Annealing major advantage over other methods is the ability to avoid becoming trapped at local minima.

An initial schedule is created by randomly scheduling the objects, and an initial cost c_{in} and temperature T_{in} are computed.

The algorithm employs a random search that not only accepts changes that decrease objective function, but also some changes that increase it.

If probability is greater than a pseudorandom number, uniformly distributed on the unity interval, the change is accepted with a probability

$$p(\Delta E) = \theta^{-\frac{\Delta E}{kT}} \tag{1}$$

where ΔE is the increase in cost (or energy) E, K is Boltzmann’s constant and T is a control parameter, which by analogy with the original application is known as the system “temperature”.

After a number of successful permutations, the temperature is decreased according the geometric cooling schedule

$$T_n = \alpha T_{n-1} \tag{2}$$

where $0 \leq \alpha < 1$ (α is the *cooling rate factor*).

It can be shown that for any given finite problem, the probability that the simulated annealing algorithm terminates with the global optimal solution approaches 1 as the annealing schedule advances.

Algorithm implementation

As previous stated, the idea is to evolve a traditional FDE RAIM approach with an innovative search engine based on the Simulated Annealing algorithm. This should allow coping with wider configuration search spaces when dealing with a higher number of satellites, typical case of multi-constellation scenarios, avoiding increasing overall computational complexity.

Simulated Annealing has been preferred to other heuristic search approaches, like evolutionary-based algorithms, due to its high computational lightness and efficiency in solving problem of such complexity.

The simplified scheme of the Simulated Annealing has been shown in Figure 1.

The objective function (to be maximized) takes the form:

$$f(c) = \alpha_1 a(c) + \alpha_2 \left(\beta_1 \frac{R_p(c)}{R_{p,\max}} + \beta_2 \frac{P_m(c)}{P_{m,\max}} \right) \tag{3}$$

where c is the current configuration, α_1, α_2, β_1 and β_2 are weights.

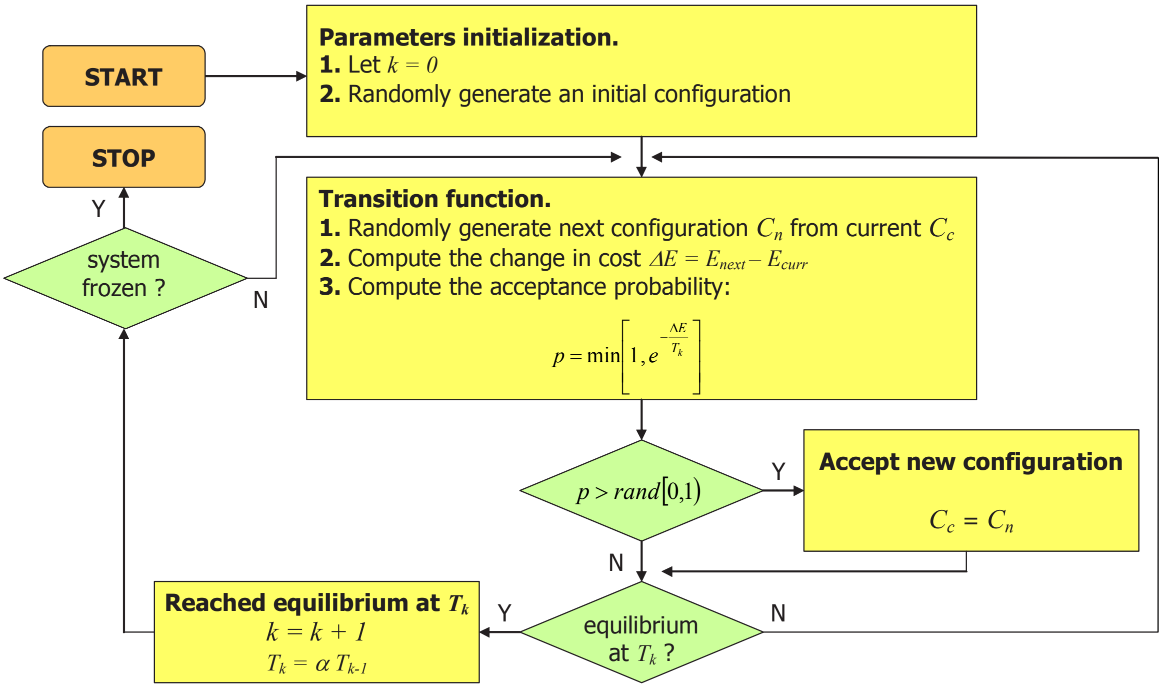


Figure 1. The Simulated Annealing algorithm

The parameter *a(c)* is an admissibility factor equal to 1 only when the configuration is admissible both from geometry screening and fault-detection points of view. In other words:

$$a(c) = \begin{cases} 1 & \text{if } R_p < R_{al} \text{ and } P_m < T_d \\ 0 & \text{otherwise} \end{cases}, \tag{4}$$

where R_p is the protection radius, R_{al} is the alarm limit, P_m is the detection statistic (obtained from the parity vector) and T_d is the detection threshold.

The admissible configurations represent the rewarding part of the whole search space, because the optimal configuration lies among them.

The objective function (3) has been properly designed in order to highly reward admissible configurations (those with *a(c)=1*), and penalizing not admissible ones.

Losing the α_l percentage of the whole reward, such solutions will be undoubtedly discarded by the optimization algorithm.

SIMULATION RESULTS

Algorithm's behaviour has been validated with a simulation framework based on the integrity parameters listed in Table 1 (NPA phase), assuming 21 mixed GPS and Galileo visible satellites (5 degrees elevation mask) according satellites' data of Table 2.

The algorithm has been stressed with three critical and quite unlikely test-cases, designed according different failure schemes: 2, 4 and 8 simultaneous satellites pseudoranges have been biased.

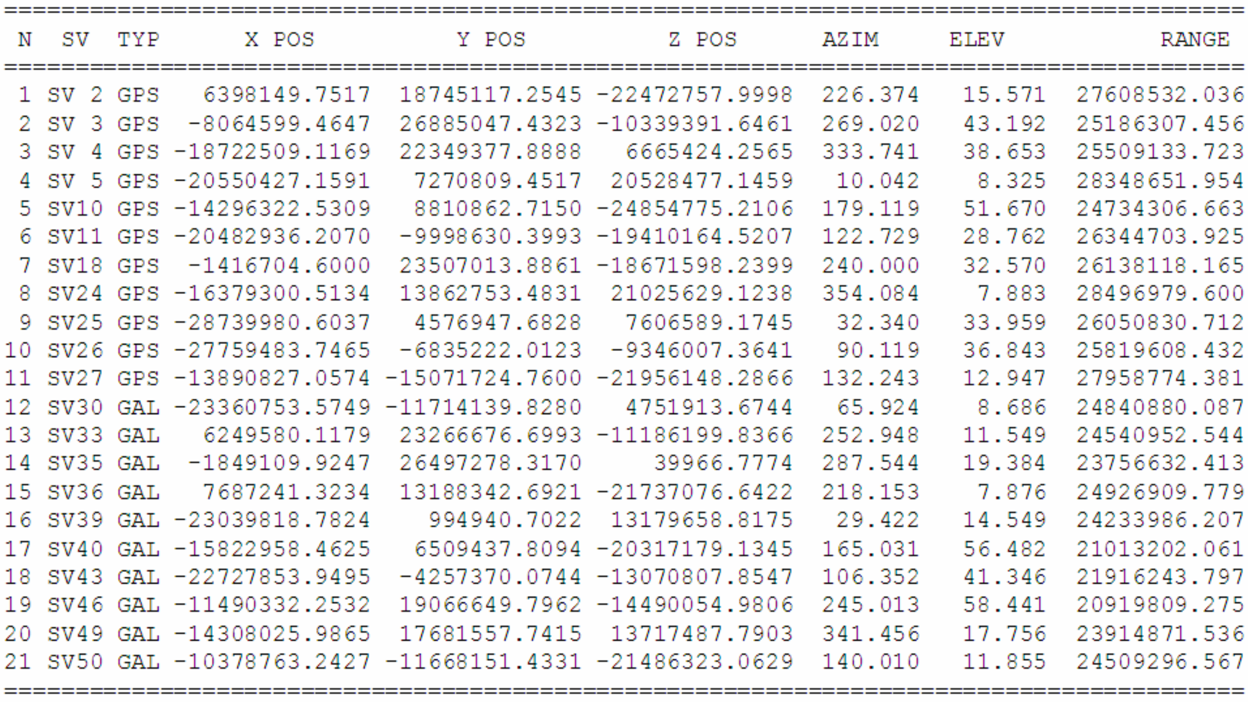
Biases entity is in the order of few hundreds meters.

In all these cases the algorithm showed good convergence capacity, providing high quality solutions in few hundred steps (avoiding the exploration of all the 54264 available configurations) and, obviously, always converging to failure-free subsets.

As in the classical problem definition, search is performed among subsets of 6 satellites.

Parameter	Integrity Parameters
Measurement noise standard deviation [m]	5
Horizontal Alarm limit [m]	555
Probability of a false detection	6.66×10^{-5}
Probability of a missed detection	0.001
Time to alarm [s]	10

Table 1. Integrity parameters



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=======================================================================================
 N  SV  TYP       X POS           Y POS           Z POS       AZIM     ELEV        RANGE
=======================================================================================
 1 SV 2 GPS   6398149.7517  18745117.2545 -22472757.9998  226.374   15.571  27608532.036
 2 SV 3 GPS  -8064599.4647  26885047.4323 -10339391.6461  269.020   43.192  25186307.456
 3 SV 4 GPS -18722509.1169  22349377.8888   6665424.2565  333.741   38.653  25509133.723
 4 SV 5 GPS -20550427.1591   7270809.4517  20528477.1459   10.042    8.325  28348651.954
 5 SV10 GPS -14296322.5309   8810862.7150 -24854775.2106  179.119   51.670  24734306.663
 6 SV11 GPS -20482936.2070  -9998630.3993 -19410164.5207  122.729   28.762  26344703.925
 7 SV18 GPS  -1416704.6000  23507013.8861 -18671598.2399  240.000   32.570  26138118.165
 8 SV24 GPS -16379300.5134  13862753.4831  21025629.1238  354.084    7.883  28496979.600
 9 SV25 GPS -28739980.6037   4576947.6828   7606589.1745   32.340   33.959  26050830.712
10 SV26 GPS -27759483.7465  -6835222.0123  -9346007.3641   90.119   36.843  25819608.432
11 SV27 GPS -13890827.0574 -15071724.7600 -21956148.2866  132.243   12.947  27958774.381
12 SV30 GAL -23360753.5749 -11714139.8280   4751913.6744   65.924    8.686  24840880.087
13 SV33 GAL   6249580.1179  23266676.6993 -11186199.8366  252.948   11.549  24540952.544
14 SV35 GAL  -1849109.9247  26497278.3170     39966.7774  287.544   19.384  23756632.413
15 SV36 GAL   7687241.3234  13188342.6921 -21737076.6422  218.153    7.876  24926909.779
16 SV39 GAL -23039818.7824    994940.7022  13179658.8175   29.422   14.549  24233986.207
17 SV40 GAL -15822958.4625   6509437.8094 -20317179.1345  165.031   56.482  21013202.061
18 SV43 GAL -22727853.9495  -4257370.0744 -13070807.8547  106.352   41.346  21916243.797
19 SV46 GAL -11490332.2532  19066649.7962 -14490054.9806  245.013   58.441  20919809.275
20 SV49 GAL -14308025.9865  17681557.7415  13717487.7903  341.456   17.756  23914871.536
21 SV50 GAL -10378763.2427 -11668151.4331 -21486323.0629  140.010   11.855  24509296.567
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Table 2. Data of the satellite scenario

This combinatorial optimization problem has proved to be very hard, since the objective function doesn't take an analytical representation and derivative information is globally unavailable.

Table 3 shows the Simulated Annealing and the objective function (equation (3)) configuration parameters used for the simulations.

Parameter	**Value**
Initial temperature T_{in}	600
Final temperature T_{fin}	1.8
Cooling rate factor α	0.85
Thermal equilibrium iterations max	5
Objective function weight α_1	0.5
Objective function weight α_2	0.5
Objective function weight β_1	0.7
Objective function weight β_2	0.3

Table 3. SA configuration parameters

Example 1. Simulations with 2 simulated failures

This is the most favourable case due to the wideness of the admissible search space (about the 54% of the whole search space). To reduce computation time, in similar cases lower values of cooling factor α are suggested.

Figure 2 shows the value of the current solution objective function (fitness) during the search process (blue curve) and the current optimal solution (red curve).

This picture shows how, frequently, the quality of the solution found at the beginning of the simulation is not high; good solutions are available approximately only from the 90th iteration. This happens because at the beginning, the annealing temperature is high, and the search is quasi-random. When the temperature is low enough, the search mainly concentrates over the admissible solutions, converging toward the optimum.

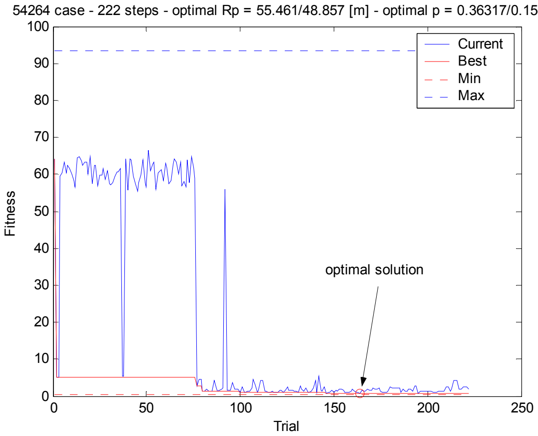

Figure 2. Example 1: optimization process

Solution found at step	164 of 222
Configurations with $R_p < R_{al}$	50841 / 54264
Configurations with $P_m < T_d$	31722 / 54264
Admissible configurations ($R_p < R_{al}$ and $P_m < T_d$)	29412 / 54264 (54%)
Satellites belonging to the optimal subset	4, 5, 24, 26, 27, 35
Biased satellites	3, 39

Table 4. Example 1: simulation results.

Example 2. Simulations with 4 simulated failures

This case is very favourable as the search space is yet large enough due to the moderate number of biases introduced. Also in this case, the optimization algorithm was able to provide a set of valid measurements discarding unhealthy ones (Figure 3).

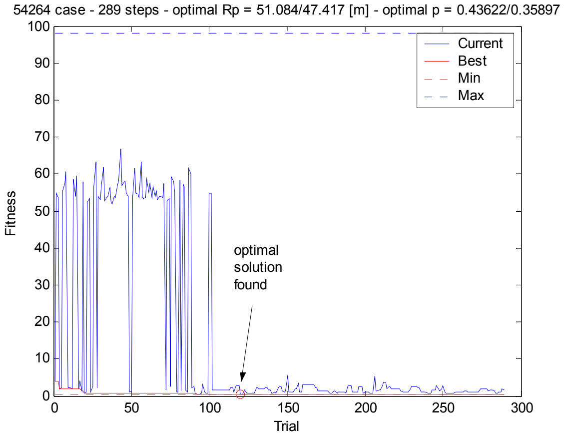

Figure 3. Example 2: optimization process

Solution found at step	120 of 289
Configurations with $R_p < R_{al}$	50841 / 54264
Configurations with $P_m < T_d$	19454 / 54264
Admissible configurations ($R_p < R_{al}$ and $P_m < T_d$)	17837 / 54264 (33%)
Satellites belonging to the optimal subset	3, 24, 25, 30, 36, 40
Biased satellites	4, 18, 33, 46

Table 5. Example 2: simulation results.

Example 3. Simulations with 8 simulated failures

In this case GPS 8 mixed GPS-Galileo faults have been simulated.

This simulation represents a realistic test-bed to validate the potential of such method, considering that the simultaneous unavailability of eight satellites represent a very conservative (as well as unrealistic) situation.

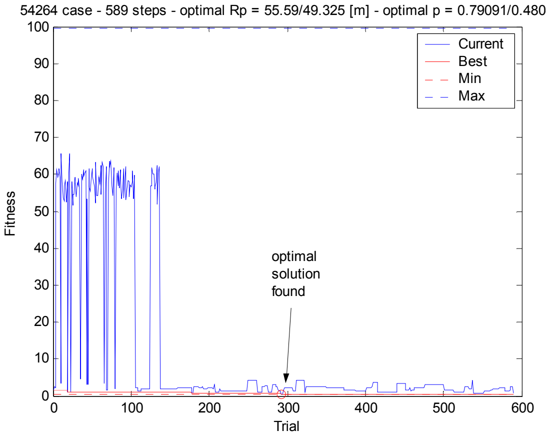

Figure 4. Example 3: optimization process

The algorithm explored 589 of 54264 configurations, finding the best solution at iteration 292.

The obtained results are judged very satisfactory, since in this case the algorithm found the eighth solution in ascending order of the objective function.

Solution found at step	292 of 589
Configurations with $R_p < R_{al}$	50841 / 54264
Configurations with $P_m < T_d$	4585 / 54264
Admissible configurations ($R_p < R_{al}$ and $P_m < T_d$)	4177 / 54264 (8%)
Satellites belonging to the optimal subset	10, 30, 35, 43, 49, 50
Biased satellites	2, 4, 18, 24, 26, 33, 39, 46

Table 6. Example 3: simulation results.

CONCLUSIONS

The proposed approach demonstrated to be applicable to many traditional and consolidated RAIM techniques based on combinatorial configuration search when applied to the multi-constellation case. When interoperating GPS and Galileo constellations the user deals with a higher number of navigation satellites. In this case, the immediate application of traditional algorithms designed for the GPS-only case is unfeasible.

An interesting feature of the shown approach is that it joins together both the geometry-screening and the error detection phases, making them totally indistinguishable.

Simulation results presented in this paper demonstrated that this solution might be a promising way to improve RAIM performances without modifying consolidated integrity theory.

Further work to be done is to investigate the behaviour of more advanced and efficient optimization approaches (e.g. adaptative simulated annealing) to further improve search performances without increasing calculus complexity.

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