Pseudo-Measurements as Aiding to INS during GPS Outages

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BIOGRAPHY

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1. INTRODUCTION

The complementary nature of INS and GPS systems can be used advantageously in navigation systems design. Numerous methods have been proposed for the integration of GPS and INS information to provide a robust navigation solution (e.g., [1], [2]).

As long as GPS measurements are available, the combined navigation solution is usually satisfactory. However, when GPS measurements are unavailable for some reason (e.g., environment type or jamming) the INS navigation solution will drift with time due to its inherent bias. A common solution to circumvent this situation is to fuse another sensor (e.g., odometer or cameras) with the INS. Yet, additional sensors require power, space and come at an increased cost. An alternative approach with a very low cost is to use knowledge of the behavior of the platform and its operating environment to aid the INS instead or in addition to actual measurements.

This type of aiding may be employed by considering the physics of the problem (platform dynamics and its operating environment) at hand and translating it to a form of pseudo-measurements. This concept was first implemented for target tracking problems by Tahk and Speyer [3]. Later, Koifman and Bar-Itzhack [4] discussed aiding of INS with aircraft dynamics equations. In ground navigation, Dissanayake et al. [5] utilized the fact that, normally, vehicles do not slip or jump off the ground as a pseudo-measurement of vehicle velocity. They used the speed encoder with the velocity pseudo-measurements to form a full measurement velocity vector. Recently, Shin [6] and Godha [7] demonstrated how to use a velocity pseudo-measurement as aiding to a linear INS error model by perturbing the velocity governing equation.

In this paper, we address the following scenario: a vehicle is traveling in an urban canyon equipped with a GPS receiver and an INS. At some point, the GPS measurements are unavailable. Our purpose is to utilize knowledge of the vehicle dynamics characteristics and the physical conditions in which it operates in order to aid the INS measurements. This is carried out by translating these conditions and characteristics into pseudo-measurements. We limit the discussion to short aiding periods, although pseudo-measurements may be useful for long time periods as well.

The rest of the paper is organized as follows: Section 2 describes the coordinate frames and the

INS error equations that will be utilized with the pseudo-measurements aiding. Section 3 presents the Kalman filter estimators which we implement for the INS aiding. Section 4 proposes several pseudo-measurement types which may be used. Section 5 presents results of case studies that demonstrate the impact of the pseudo-measurements on the navigation solution accuracy. Section 6 presents the conclusions.

2. INS ERROR EQUATIONS

The following coordinate frames are used in this work: Inertial frame (i-frame), Earth Centered Earth Fixed (e-frame) frame, North-East-Down (NED) frame (n-frame) and Body frame (b-frame). The i-frame origin is at the center of the Earth. The x-axis points towards the mean Vernal equinox, the z-axis is parallel to the Earth spin axis and the y-axis completes a right handed orthogonal frame. The e-frame has its origin at the center of the Earth and rotates with the Earth spin. The x-axis points towards the Greenwich meridian, the z-axis is parallel to the Earth spin axis and the y-axis completes a right handed orthogonal frame. The n-frame has its origin fixed on the earth surface at the initial latitude/longitude of the vehicle. The x-axis points towards the geodetic north, the z-axis is on the local vertical pointing down and the y-axis completes a right handed orthogonal frame. The b-frame origin is at the vehicle center of mass. The x-axis is parallel to the vehicle longitudinal symmetry axis pointing forward, the z-axis points down and the y-axis completes a right handed orthogonal frame.

Raw measurements from accelerometers and gyros are measured along the b-frame. They are transformed to the n-frame, where data integration is performed. The position in the n-frame is expressed by curvilinear coordinates $r^n = \begin{bmatrix} \phi & \lambda & h \end{bmatrix}^T$ where, ϕ is the latitude, λ is the longitude and h is the height above the Earth surface. The equations of motion in the n-frame are given by [2]:

$$\begin{bmatrix} \dot{r}^n \\ \dot{v}^n \\ \dot{T}^{b\to n} \end{bmatrix} = \begin{bmatrix} D^{-1}v^n \\ T^{b\to n} f^b + g_1^n - \left(2\omega_{ie}^n + \omega_{en}^n\right)\times v^n \\ T^{b\to n}\Omega_{nb}^b \end{bmatrix} \quad (1)$$

$$D^{-1} = \begin{bmatrix} \frac{1}{(M+h)} & 0 & 0 \\ 0 & \frac{1}{(N+h)\cos(\phi)} & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (2)$$

where $v^n = \begin{bmatrix} v_N & v_E & v_D \end{bmatrix}$ is the vehicle velocity, $T^{b\to n}$ and $T^{n\to b}$ are the transformation matrices from the b-frame to the n-frame and vice-versa, respectively. f^b is the measured specific force, ω_{ie}^n is the Earth rate with respect to the i-frame, ω_{en}^n is the turn rate of the n-frame with respect to the Earth, g_1^n is local gravity vector, M and N are the radii of curvature in the meridian and prime vertical, respectively and Ω_{nb}^b is the skew-symmetric form of the body rate with respect to the n-frame given by:

$$\omega_{nb}^b = \omega_{ib}^b - T^{n\to b}\left(\omega_{ie}^n + \omega_{en}^n\right) \quad (3)$$

The INS mechanization equations provide no information about errors in the system states (caused by measurement errors) as they process raw data from the Inertial Measurement Unit (IMU) to estimate navigation parameters. The IMU outputs contain additional errors that cannot be compensated for. These errors are due to uncertainties in the sensors such as spurious magnetic fields and temperature gradients. Thus, to improve the performance of the INS it is necessary to develop an error model which describes how the IMU sensor errors propagate through the equation of motion (Eq. (1)) into navigation errors. These navigation errors are then corrected for in order to obtain corrected navigation solution.

Several models (e.g. [10], [13] and [2]) were developed to describe the time-dependent behavior of these errors. A classic approach is the perturbation analysis, in which navigation parameters are perturbed with respect to the true n-frame. This is done by taking a first order Taylor series expansion of the states in Eq. (1). A complete derivation of this model can be found for example in [14], [6] and [15]. The state-space model is given by:

$$\begin{bmatrix} \delta\dot{r}^n \\ \delta\dot{v}^n \\ \dot{\varepsilon}^n \\ \delta\dot{b}_a \\ \delta\dot{b}_g \end{bmatrix} = F \begin{bmatrix} \delta r^n \\ \delta v^n \\ \varepsilon^n \\ \delta b_a \\ \delta b_g \end{bmatrix} + G \begin{bmatrix} \upsilon_a \\ \upsilon_g \\ \upsilon_{ba} \\ \upsilon_{bg} \end{bmatrix} \quad (4)$$

$$F=\begin{bmatrix} F_{rr} & F_{rv} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ F_{rv} & F_{vv} & f^n & T^{b\to n} & 0_{3\times3} \\ F_{er} & F_{ev} & -\Omega_{in}^n & 0_{3\times3} & -T^{b\to n} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \left(-1/\tau_a\right)_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \left(-1/\tau_g\right)_{3\times3} \end{bmatrix} \quad (5)$$

$$G=\begin{bmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ T^{b\to n} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & -T^{b\to n} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix} \quad (6)$$

where the state vector consists of position error, velocity and attitude errors and accelerometer and gyro bias/drift. A detailed description of the parameters in Eq. (4) is given in an appendix. Eq. (4) defines the system dynamics for the linear Kalman filter, which is introduced in the next section.

3. LINEAR KALMAN FILTER

We incorporate the INS dynamics with pseudo-measurements aiding in a linear Kalman filter. In general, a Kalman filter algorithm contains two steps: prediction of the state based on the system model, and update of the state based on the measurements. The first step in the Kalman filter is prediction of the state and its associated covariance [1]:

$$\hat{x}_{k+1}^- = \Phi \hat{x}_k^+ , \Phi = e^{F(t)\Delta t} \quad (7)$$

$$P_{k+1}^- = \Phi P_k^+ \Phi^T + Q_k \quad (8)$$

where the superscripts – and + represent the predicted quantity (before measurement update) and the updated quantity (after measurement update). x and P are the system state and the associated error covariance matrix, respectively. Φ is the state transition matrix from time k to time $k+1$, $F(t)$ is the system dynamic matrix and Q_k is the process noise covariance matrix [11] given by:

$$Q_k \approx \frac{1}{2}\left[\Phi_k G(t_k) Q(t_k) G^T(t_k) + G(t_k) Q(t_k) G^T(t_k) \Phi_k^T\right]\Delta t \quad (9)$$

where, $G(t)$ is the shaping matrix. Δt is the time step.

The second step is the measurement update:

$$K_{k+1} = P_{k+1}^- H_{k+1}^T \left(H_{k+1} P_{k+1}^- H_{k+1}^T + R_{k+1}\right)^{-1} \quad (10)$$

$$\hat{x}_{k+1}^+ = \hat{x}_{k+1}^- + K_{k+1}\left(z_{k+1} - H_{k+1}\hat{x}_{k+1}^-\right) \quad (11)$$

$$P_{k+1}^+ = \left(I - K_{k+1} H_{k+1}\right) P_{k+1}^- \quad (12)$$

where K_k is the Kalman gain, H_k is the measurement matrix, R_k is the measurement noise covariance matrix and z_k is the measurement.

4. PSEUDO-MEASUREMENTS

Pseudo-measurements take advantage of knowledge of the vehicle dynamics and physical conditions the vehicle experiences, and utilize them as measurements in the vehicle state estimation process. Pseudo-measurements do not involve actual sensors and so the cost associated with their introduction in the model is very low. Furthermore, since pseudo-measurements are always available, their update rate can be set to the INS operating bandwidth to ensure continues measurement update. We divide the pseudo-measurements in two categories: those expressing vehicle dynamics and those describing the vehicle operating environment.

4.1 PSEUDO-MEASUREMENTS USING VEHICLE DYNAMICS

4.1.1 BODY VELOCITY

Vehicles travel on the ground and do not slide on it. Thus, we can assume that in the b-frame the velocities in the y_B and z_B directions are almost zero ([5], [6]), namely $v_{B_y} \cong 0$ and $v_{B_z} \cong 0$. Under these assumptions, the computed velocity in the b-frame can be expressed as

$$v_B = \left(T^{b\to n}\right)^T v_N \quad (13)$$

After perturbing Eq. (13) and rearranging it, we obtain

$$\delta v_B = T^{n\to b}\delta v_N - T^{n\to b}\left(v_N \times\right)\delta\varepsilon_N \quad (14)$$

where $\left(v_N \times\right)$ is the skew symmetric form of the velocity vector. From the second and third rows of the pervious vector equation, the measurement equations can be constructed as:

$$z_k = \begin{bmatrix} v_{B_y} \\ v_{B_z} \end{bmatrix}_{INS} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (15)$$

$$H_k = \begin{bmatrix} 0_{1\times3} & T_{12}^{b\to n} & T_{22}^{b\to n} & T_{32}^{b\to n} & v_E T_{32}^{b\to n} - v_D T_{22}^{b\to n} & v_D T_{12}^{b\to n} - v_N T_{32}^{b\to n} & v_N T_{22}^{b\to n} - v_E T_{12}^{b\to n} & 0_{1\times6} \\ 0_{1\times3} & T_{13}^{b\to n} & T_{23}^{b\to n} & T_{33}^{b\to n} & v_E T_{33}^{b\to n} - v_D T_{23}^{b\to n} & v_D T_{13}^{b\to n} - v_N T_{33}^{b\to n} & v_N T_{23}^{b\to n} - v_E T_{13}^{b\to n} & 0_{1\times6} \end{bmatrix}$$
(16)

Eqs. (15) and (16) are used as inputs to the Kalman filter Eqs. (10)-(12).

4.1.2 BODY ANGULAR VELOCITY

In the general case a platform can have body angular velocities in all three directions: $\omega_{ib} = \begin{bmatrix} p & q & r \end{bmatrix}^T$. However, as noted above the vehicle travels on the ground, and so $p = 0$ and $q = 0$. That is, the vehicle can only change its yaw (heading) angle. The corresponding measurement can be expressed by:

$$z = \left[\omega_{ib}\right]_{INS} - \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix} \quad (17)$$

The first two rows of Eq. (17) may be used as pseudo-measurements. However, the body angular velocity is not modeled as a state in the system dynamics (Eq. (4)), only its bias. We can add the body angular velocity as a new state or convert the state measurement in Eq. (17) to some measurement on the body angular velocity bias. In order to keep the state-space model simple, we concentrate on the second option. That is, the corresponding assumption on the biases are $\delta b_{xg} = 0$ and $\delta b_{yg} = 0$ since there should be no angular velocity in these directions**.** The resulting measurement equations are given by:

$$z = \begin{bmatrix} \delta b_{xg\,INS} - 0 \\ \delta b_{yg\,INS} - 0 \end{bmatrix} \quad (18)$$

$$H = \begin{bmatrix} 0_{1\times9} & 1 & 0_{1\times5} \\ 0_{1\times10} & 1 & 0_{1\times4} \end{bmatrix} \quad (19)$$

4.1.3 BODY ACCELERATION

Similar to the body velocity assumption, the body acceleration in the b-frame y_B and z_B directions is almost zero. Thus, it may be assumed that $a_{B_y} = 0$ and $a_{B_z} = 0$. However, this assumption does not hold when the vehicle is turning since the body acceleration has a component in the y-axis due to the centrifugal force. Therefore, we can only use the z-axis component. The corresponding measurement is given by:

$$z = \left[a_{zb}\right]_{INS} - 0 \quad (20)$$

As with the body velocity, the acceleration is not modeled as a state, only its bias. As with the body angular velocity, we translate the state measurement into a bias measurement. That is, we assume $\delta b_{za} = 0$ **.** The equivalent measurement equations are given by:

$$z = \delta b_{za\,INS} - 0 \quad (21)$$

$$H = \begin{bmatrix} 0_{1\times14} & 1 \end{bmatrix} \quad (22)$$

4.2 PSEUDO-MEASUREMENTS FOR VEHICLE OPERATING ENVIRONMENT

4.2.1 CONSTANT HEIGHT

Usually, when driving in an urban environment for short time periods the height remains almost constant. This information can be used as pseudo-measurement. It holds approximately when the vehicle is traveling in a region with an almost constant height.. This type of pseudo-measurement was implemented [7] with respect to the e-frame. This means that the constant height pseudo-measurement was transformed from a single position component in the n-frame to a full position vector measurement in the e-frame. Thus, when the pseudo-measurement is incorrect, the whole e-frame position vector is incorrect. In contrast, in the n-frame errors in the pseudo-measurement do not affect the whole position vector. When assuming constant height $h = h_c$ the measurement equations can be constructed as:

$$z = h_{INS} - h_c \quad (23)$$

$$H = \begin{bmatrix} 0 & 0 & 1 & 0_{1\times12} \end{bmatrix} \quad (24)$$

4.2.2 CONSTANT LLH POSITION

As with the constant height aiding, we can assume that the latitude and longitude of the vehicle, driving in an urban environment for short time periods, are also almost constant. In order to obtain a full position measurement, the constant latitude, longitude and height assumptions are combined in the following manner:

$$z = \begin{bmatrix} \phi_{INS} - \phi_c \\ \lambda_{INS} - \lambda_c \\ h_{INS} - h_c \end{bmatrix} \quad (25)$$

$$H = \begin{bmatrix} I_3 & 0_{3\times 12} \end{bmatrix} \quad (26)$$

This pseudo-measurement states that the vehicle is still (which is most likely not the case). However, considering short time periods, the distance the vehicle passes cannot be large and therefore, the measurement noise compensate for the vehicle motion.

4.3 COMBINING PSEUDO-MEASUREMENTS

Each of the pseudo-measurements can be used as a standalone aiding or can be combined with any other pseudo-measurement aiding. Since several pseudo-measurements work well in improving the position accuracy, and others work well in improving the velocity accuracy, it is very natural combining them.

5. CASE-STUDIES

We present three case-studies to demonstrate the usefulness of the proposed pseudo-measurements. In the first, the road is straight and leveled. In the second, we introduce an undulating road. In the third scenario the road is on a slope leading to a change in height. All scenarios assume that the "true" trajectory is that of a stationary vehicle accelerating for five seconds, and maintaining a constant velocity afterwards until the end of the scenario.

We implemented the 15-state filter given in Eq. (4). For each scenario Monte-Carlo simulations with 100 replications were made. For each case-study a MEMS INS, whose parameters are given in Table 1, was implemented.

Table 1: Simulation parameters for MEMS INS [7]

Velocity Random Walk	$280 \cdot 10^{-6}$ $\left[\left(m/s^2\right)/\sqrt{Hz}\right]$	
Angular Random Walk	$8.7 \cdot 10^{-4}$ $\left[\left(rad/s\right)/\sqrt{Hz}\right]$	
Gyro bias	$\sigma = 0.001\left[rad/s\right]$	$T = 375\left[\sec\right]$
Acceleration bias	$\sigma = 0.009\left[m/s^2\right]$	$T = 227\left[\sec\right]$

To evaluate the contribution of the various pseudo-measurements, the following error measures are utilized:

$$\varepsilon_h(t) = h_{aiding}(t) - h_{nominal}(t) \quad (27)$$

$$\varepsilon_{ll}^{RMS}(t) = \sqrt{\varepsilon_{lat}^2(t) + \varepsilon_{long}^2(t)} \quad (28)$$

$$\varepsilon_{vel}^{RMS}(t) = \sqrt{\varepsilon_{vn}^2(t) + \varepsilon_{ve}^2(t) + \varepsilon_{vd}^2(t)} \quad (29)$$

where $\varepsilon_h(t)$ is the height error, $h_{nominal}(t)$ and $h_{aiding}(t)$ are the true height and the estimated height obtained from the pseudo-measurements aiding, respectively. $\varepsilon_{ll}^{RMS}(t)$ and $\varepsilon_{vel}^{RMS}(t)$ are the root mean square error of the latitude and longitude, and of the NED velocity components, respectively. $\varepsilon_{lat}(t)$ and $\varepsilon_{long}(t)$ are the latitude and the longitude errors, respectively. $\varepsilon_{vn}(t)$, $\varepsilon_{ve}(t)$ and $\varepsilon_{vd}(t)$ are the north, east and down velocity errors, respectively. These errors are defined in the same manner as in Eq. (27).

We use separate error measures for the height, latitude/longitude and velocity as different applications may find interest in different measures.

For each of the following scenarios all pseudo-measurements described in Section 4 were applied as aiding to the INS. Several combinations of these pseudo-measurements were also utilized, including: i) body angular velocity with body velocity, ii) body angular velocity with constant LLH, iii) body velocity with constant LLH.

5.1 SCENARIO I

In this scenario, a stationary vehicle accelerates to $v = 45\left[km/h\right]$ by applying a constant acceleration of $a = 2.5\left[m/s^2\right]$ for five seconds. Afterwards the vehicle maintains a constant velocity until the end. The simulated error measures of the INS mechanization solution is presented in Tables 2-4 for height, lat/long and velocity errors, respectively. In these tables and in the rest to follow we present only the INS-only solution and three types of pseudo-measurements aiding: The body velocity, which has been previously proposed in [5], body angular velocity (Section 4.1.2) and constant LLH (Section 4.2.2). Focus on the last two is due to their overall best performance compared to other aiding types. The tables also show percent reduction in the RMS errors compared to the standalone INS solution.

Table 2: Scenario 1: Height error [m] mean and standard deviation.

Aiding Type	**T=60 [sec]**	**T=120 [sec]**	**T=180 [sec]**
INS only [m]	8.5	143	974
Body Velocity [m]	7±10 (-17.6%)	9±7 (-93.7%)	67±11 (-93.1%)
Body Angular Velocity [m]	2±0.3 (-99.8%)	118±3 (-17.5%)	793±10 (-18.6%)
Constant LLH[m]	0.02± 1.8E-5 (-99.8%)	0.1± 1.7E-5 (-99.9%)	0.16± 1.5E-5 (-99.9%)

Table 3: Scenario 1: Velocity RMS error [m/s] mean and standard deviation.

Aiding Type	**T=60 [sec]**	**T=120 [sec]**	**T=180 [sec]**
INS only [m/s]	17	90	232
Body Velocity [m/s]	11±5 (-32.3%)	11.6±1.5 (-87.1%)	15±1 (-93.5%)
Body Angular Velocity [m/s]	0.5± 0.8 (-97.0%)	5.1±2.3 (-94.3%)	22.4±3.7 (-90.3%)
Constant LLH[m/s]	12.5± 4.3E-3 (-26.5%)	12.6± 4.8E-3 (-86.0%)	12.5± 4.5E-3 (-94.6%)

Table 4: Scenario 1 Lat/Long RMS error [rad] mean and standard deviation.

Aiding Type	**T=60 [sec]**	**T=120 [sec]**	**T=180 [sec]**
INS only [rad]	4.797E-5	4.958E-4	1.980E-3
Body Velocity [rad]	4.740E-5± 9.7E-6 (-1.2%)	1.926E-4± 1.9E-5 (-61.2%)	3.192E-4± 1.9E-5 (-83.9%)
Body Angular Velocity [rad]	1.079E-6± 3.0E-6 (-97.7%)	7.018E-5± 1.7E-5 (-85.8%)	2.810E-4± 4.6E-5 (-85.8%)
Constant LLH [rad]	1.209E-4± 1E-10 (-26.4%)	2.465E-4± 1E-10 (-50.2%)	3.707E-4± 1E-10 (-94.6%)

Table 2 shows that an INS solution with no aiding has a height error of $974[m]$ after $180[\sec]$. All of the pseudo-measurements improved the standalone INS solution by between 10% and 99%. The constant height pseudo-measurements aiding has obtained the best performance lowering the height error to only $0.02[m]$, i.e., eliminated 99.99% of the height error. The velocity RMS of the unaided INS was $232[m/s]$ after $180[\sec]$. The constant latitude, longitude and height aiding result was less than $12.5[m/s]$ at the same stage. The latitude/longitude RMS error was also improved by up to 94% using the same aiding. Results show that in this case-study where ideal conditions exist, most pseudo-measurement aiding, including those which are not listed in the tables, improved over the standalone INS solution by over 80% of the RMS errors throughout the examined scenario. In addition, until $60[\sec]$ body angular velocity pseudo measurement has also improved the body velocity pseudo measurement by more than 70%. The overall best performance of all states was obtained by constant latitude, longitude and height aiding. Yet, for the velocity and latitude/longitude only, the body angular velocity obtained the best performance.

5.2 SCENARIO II

The difference between this scenario and the previous one is the geometry (topography) of the road which was leveled first and now is undulating. As a result, the height of the trajectory is no longer constant. The road undulation is modeled by colored noise which results in height differences up to a meter along the trajectory. The results of the error measures for scenario II are listed in Tables 5-7.

Table 5: Scenario 2: Height error [m] mean and standard deviation.

Aiding Type	**T=60 [sec]**	**T=120 [sec]**	**T=180 [sec]**
INS only[m]	5.57	131.8	756
Body Velocity [m]	5.21±14 (-6.5%)	11.59±21 (-91.2%)	118±13 (-84.4%)
Body Angular Velocity [m]	3.02±0.3 (-30.6%)	123±1.9 (-6.7%)	702±2.1 (-7.1%)
Constant LLH [m]	0.3± 1.9E-5 (-94.6%)	0.78± 1.5E-5 (-94.4%)	0.98± 1.6E-5 (-99.9%)

The undulating road introduces small accelerations in the z-axis of the body frame. Consequently, the underlying assumptions of constant height (Section

4.2.1), constant slope (Section 4.2.3) and body velocity (Section 4.1.1), are no longer exact.

As a result, the body acceleration pseudo measurement (Section 4.1.3) does not improve the accuracy of the standalone INS (less than 1%). Nevertheless, as in the pervious scenario, all other pseudo-measurement aiding types improved the standalone INS solution. Improvement rate is between 5%-99%. In particular, the constant height pseudo-measurement (Section 4.2.1) lowers the error by almost two orders of magnitude after 180sec, even though the constant height assumption is violated. This result implies that the constant height pseudo-measurement can perform well even if height along the trajectory varies slightly. Tables 6-7 show that the body angular velocity pseudo-measurement has the best performance. It reduces the velocity error to less then 1[m/s] after 120sec from almost 60[m/s] of the standalone INS. Notice also that both body angular velocity outperformed the body velocity pseudo-measurement by more than 40%.

Table 6: Scenario 2: Velocity RMS error [m/s] mean and standard deviation.

Aiding Type	T=60 [sec]	T=120 [sec]	T=180 [sec]
INS only [m/s]	13.8	58.6	156.7
Body Velocity [m/s]	11.4± 2.4 (-17.4%)	15.1±4.3 (-74.2%)	18.7±0.6 (-88.0%)
Body Angular Velocity [m/s]	0.38±0.9 (-97.2%)	0.89±2.8 (-98.5%)	10.9±5.4 (-93.0%)
Constant LLH [m/s]	6.2±2.5 (-55.0%)	15.1±1.3 (-74.2%)	17.7±0.8 (-88.7%)

Table 7: Scenario 2 Lat/Long RMS error [rad] mean and standard deviation.

Aiding Type	T=60 [sec]	T=120 [sec]	T=180 [sec]
INS only [rad]	3.76E-5	3.42E-4	1.24E-3
Body Velocity [rad]	3.87E-4± 5.7E-6 (+9.3%)	2.06E-4± 1.7E-5 (-39.7%)	3.34E-4± 2E-5 (-73.0%)
Body Angular Velocity [rad]	1.54E-6± 3.5E-6 (-95.9%)	7.05E-6± 2E-5 (-97.9%)	1.67E-5± 5.9E-5 (-98.6%)
Constant LLH [rad]	1.2E-5± 1E-10 (-68.0%)	2.46E-5± 1E-10 (-92.8%)	3.71E-5± 1E-10 (-97.0%)

5.3 SCENARIO III

In this scenario, the vehicle travels on a road with a constant 5° slope. Some assumptions that were set as the basis for the pseudo-measurements (namely constant height) are not exact any longer. Results for scenario III are listed in Tables 8-10.

Table 8: Scenario 3: Height error [m] mean and standard deviation.

Aiding Type	T=60 [sec]	T=120 [sec]	T=180 [sec]
INS only [m]	12.64	346.7	2933
Body Velocity [m]	12.52± 10 (-0.94%)	58.5± 12.4 (-83.1%)	137±13 (-95.3%)
Body Angular Velocity [m]	2.4±0.3 (-81.0%)	270±2 (-22.1%)	2600±12 (-11.3%)
Constant LLH [m]	12.43± 2E-4 (-1.6%)	129± 2E-5 (-62.7%)	195± 1.3E-5 (-93.3%)

Table 8 show that an INS solution with no aiding has a height error of $2933[m]$ after $180[\sec]$ which was lowered by the body $137[m]$. However, for time periods less than $60[\sec]$ the body angular velocity aiding obtained the best performance. As expected, several pseudo-measurements, such as constant height (Section 4.2.1), did not perform well since the underlying assumption of their derivation was violated. Yet, all other pseudo-measurements improved the standalone INS solution by 10%-99%. Tables 9 and 10 shows that the best results were obtained by the body angular velocity pseudo-measurement, particularly within the first $60[\sec]$ interval, where amount of improvement is ~95%. In addition, the constant latitude, longitude and height aiding has obtained best result for the velocity RMS after $180[\sec]$. Moreover, its velocity RMS error was nearly constant regardless to time.

Table 9: Scenario 3: Velocity RMS error [m/s] mean and standard deviation.

Aiding Type	T=60 [sec]	T=120 [sec]	T=180 [sec]
INS only [m/s]	18.1	138.9	422.7
Body Velocity [m/s]	10.1± 1 (-44.2%)	17.3±0.6 (-87.5%)	53.9±0.2 (-87.2%)
Body Angular	0.97±0.6 (-94.6%)	14.2±1.7 (-89.8%)	86±3.8 (-79.6%)

Velocity [m/s]			
Constant LLH [m/s]	12.6±2 (-30.4%)	12.5±1.2 (-90.9%)	12.6±0.6 (-97.0%)

Table 10: Scenario 3 Lat/Long RMS error [rad] mean and standard deviation.

Aiding Type	T=60 [sec]	T=120 [sec]	T=180 [sec]
INS only [rad]	3.987E-5	6.756E-4	3.165E-3
Body Velocity [rad]	4.014E-5± 6.5E-6 (+0.6%)	2.090E-4± 1.1E-5 (-69.0%)	3.227E-4± 1 E-5 (-89.8%)
Body Angular Velocity [rad]	1.040E-6± 2.4E-6 (-97.4%)	2.417E-5± 1.2E-5 (-96.4%)	1.644E-4± 3.7E-5 (-94.8%)
Constant LLH [rad]	3.799E-5± 1.2E-5 (-4.7%)	2.456E-4± 1.9E-5 (-63.57%)	3.694E-4 ±2.6E-5 (-88.3%)

6. CONCLUSIONS

In this paper, we proposed several pseudo-measurements as aiding for MEMS INS in short periods of GPS outage (up to 3-minutes). The proposed pseudo-measurements were examined through simulations of several characteristic driving scenarios. These were designed so that the underlying assumptions of several of the pseudo-measurements were not exact in order to examine their robustness.

Results show that in most cases, introduction of the pseudo-measurements greatly reduces the navigation errors obtained with the standalone INS. A general observation is that pseudo-measurements for vehicle dynamics (Section 4.2) outperform those for the vehicle operating environment (Section 4.1). This behavior can be attributed to the assumptions made on the vehicle operating environment, which are easily violated (e.g., height change) whereas assumptions on the vehicle dynamics are more robust.

The analysis shows that the best performance for all three error measures examined (height, velocity, and latitude/longitude) is obtained with the body angular velocity and the constant latitude/longitude and height pseudo-measurements. On the other hand, body angular velocity pseudo-measurement achieved the most significant improvement of the standalone INS errors in the velocity and latitude/longitude error measures.

Results therefore show that without traditional ('external") aiding, INS navigation errors can be confined into small values by incorporation of appropriate pseudo-measurements which is made with little computational overhead.

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8. APPENDIX

The following matrixes are associated with the INS state space error model Eq. (4)

$$F_{rr} = \begin{bmatrix} 0 & 0 & \frac{-V_N}{(M+h)^2} \\ \frac{V_E \sin(\phi)}{(N+h)\cos^2(\phi)} & 0 & \frac{-V_E \sin(\phi)}{(N+h)^2 \cos^2(\phi)} \\ 0 & 0 & 0 \end{bmatrix} \quad \text{(A.1)}$$

$$F_{rv} = \begin{bmatrix} \frac{1}{(M+h)} & 0 & 0 \\ 0 & \frac{1}{(N+h)\cos(\phi)} & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad \text{(A.2)}$$

$$F_{vr} = \begin{bmatrix} -2V_E\omega_e \cos(\phi) - \frac{V_E^2}{(N+h)\cos^2(\phi)} & 0 & \frac{-V_N V_D}{(M+h)^2} + \frac{V_E^2 \tan(\phi)}{(N+h)^2} \\ 2\omega_e\left(V_N \cos(\phi) - V_D \sin(\phi)\right) + \frac{V_E V_N}{(N+h)\cos^2(\phi)} & 0 & \frac{-V_N V_D}{(N+h)^2} - \frac{V_E V_N \tan(\phi)}{(N+h)^2} \\ 2V_E\omega_e \sin(\phi) & 0 & \frac{V_E^2}{(N+h)} + \frac{V_N^2}{(M+h)} - \frac{2\gamma}{(R+h)} \end{bmatrix}$$

(A.3)

$$F_{vv} = \begin{bmatrix} \frac{V_D}{(M+h)} & -2\omega_e \sin(\phi) - \frac{2V_E \tan(\phi)}{(N+h)} & \frac{V_N}{(M+h)} \\ 2\omega_e \sin(\phi) + \frac{V_E \tan(\phi)}{(N+h)} & \frac{V_D + V_N \tan(\phi)}{(N+h)} & 2\omega_e \cos(\phi) + \frac{2V_E}{(N+h)} \\ -\frac{2V_N}{(M+h)} & -2\omega_e \cos(\phi) - \frac{2V_E}{(N+h)} & 0 \end{bmatrix}$$

(A.4)

$$F_{\varepsilon r} = \begin{bmatrix} -\omega_e \sin(\phi) & 0 & \frac{-V_E}{(N+h)^2} \\ 0 & 0 & \frac{V_N}{(M+h)^2} \\ -\omega_e \cos(\phi) - \frac{V_E}{(N+h)\cos^2(\phi)} & 0 & \frac{V_E \tan(\phi)}{(N+h)^2} \end{bmatrix} \quad \text{(A.5)}$$

$$F_{\varepsilon v} = \begin{bmatrix} 0 & \frac{1}{(N+h)} & 0 \\ -\frac{1}{(M+h)} & 0 & 0 \\ 0 & \frac{-\tan(\phi)}{(N+h)} & 0 \end{bmatrix} \quad \text{(A.6)}$$

$$F_{\varepsilon\varepsilon} = \begin{bmatrix} 0 & \omega_e \sin(\phi) + \frac{V_E \tan(\phi)}{(N+h)} & \frac{V_N}{(M+h)} \\ -\omega_e \sin(\phi) - \frac{V_E \tan(\phi)}{(N+h)} & 0 & -\omega_e \cos(\phi) - \frac{V_E}{(N+h)} \\ -\omega_e \cos(\phi) - \frac{V_E}{(N+h)\cos^2(\phi)} & \omega_e \cos(\phi) + \frac{V_E}{(N+h)} & 0 \end{bmatrix}$$

(A.7)

where $v^n \triangleq \begin{bmatrix} v_N & v_E & v_D \end{bmatrix}^T$ is the velocity vector in the n-frame and the rest of the parameters were defined in the text.