A Precise Point Positioning Ambiguity Resolution Method with Narrow-Lane Ambiguity Fractional Bias Eliminated

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BIOGRAPHY

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ABSTRACT

Ambiguity resolution with PPP (Precise Point Positioning) aims at reducing the convergence time of the float solutions and improving its obtainable positioning accuracy. This paper describes a PPP ambiguity resolution procedure with the narrow-lane ambiguity fractional bias eliminated based on a functional model solely using single-difference between-satellites ionospheric-free carrier phase observations. The first step of the procedure is to estimate ionospheric-free float ambiguities expressed in wide-lane and narrow-lane ambiguity terms. The second step is to pseudo-fix the wide-lane ambiguity by differencing the narrow-lane code and the wide-lane carrier phase measurements. Each pseudo-fixed ambiguity has an integer offset to the true wide-lane ambiguity introduced by the satellite hardware bias from the narrow-lane code and wide-lane carrier phase observations. The third step is to replace the true wide-lane ambiguity component in the ionospheric-free combined ambiguity (from step 1) with the pseudo-fixed ambiguity (from step 2) to achieve a pseudo narrow-lane float ambiguity. When compared to the true narrow-lane ambiguity, this pseudo narrow-lane ambiguity has an integer offset and fractional bias, namely narrow-lane ambiguity offset and narrow-lane ambiguity fractional bias, which are introduced by the wide-lane integer ambiguity offset, satellite hardware bias from the narrow-lane and wide-lane carrier phase combinations. In the fourth step, a PPP user can eliminate the narrow-lane ambiguity fractional bias from the pseudo narrow-lane float ambiguity using correction generated by a reference network. Using this method, the integer property of narrow-lane ambiguity can be recovered and ambiguity resolution can then be conducted.

INTRODUCTION

Precise Point Positioning (PPP) was initially proposed to save computational cost for GPS network data process [*Ge, et al.*, 2008; *Zumberge, et al.*, 1997]. PPP has attracted an broad interest due to its features of operational simplicity, cost-effectiveness, no base station setup requirement and providing comparable positioning accuracy to double-difference approach [*Chen,* 2004; *Gao and Shen*, 2001; *Kouba and Heroux,* 2001].

A wide adoption of PPP especially for real-time applications however is limited by a major factor, namely, its long convergence time which is typically fifteen minutes to half an hour or even longer before its position accuracy could reach decimeter level. Techniques for ambiguity resolution with PPP therefore should be investigated. Some research results have already published in literatures. In *Ge, et al.* (2008), a method is proposed which is first to separate the ionospheric-free float ambiguities in wide- and narrow-lane and then to implement PPP ambiguity resolution. The method has two potential problems. First, the effective wavelength of the narrow-lane ambiguity in this method is about 5.35 cm which is only half of the regular narrow-lane wavelength so it makes PPP ambiguity resolution very difficult. Second, a fractional bias, namely Un-calibrated Phase Delays (UPD), needs to be calibrated before PPP ambiguity resolution can be conducted, but the origin of such kind UPD has not been clearly explained in the paper. *Collins*, (2008a) and *Collins, et al.* (2008b) proposed a decoupled clock model for ambiguity resolution with PPP, where the common-oscillator hardware biases which are the actual cause for the PPP ambiguity fractional bias are considered. But their properties such as their time-variation have not been investigated. Similar to Ge et al. (2008), the method also separates ionospheric-free float ambiguities in wide- and narrow-lane. *Laurichesse and Mercier* (2007) proposed to separate ionospheric-free float ambiguities in wide-lane and L1. The significance in this research is the effective wavelength of L1 ambiguity is equal to the regular narrow-lane wavelength, which is doubled compared to other methods. Its mathematical model is based on un-differenced observations but the instability of the receiver hardware bias has not been considered.

In this paper, GPS un-differenced measurements are firstly rationalized by introducing the receiver and satellite hardware biases. The stabilities of these two biases are then briefly discussed based on previous research work. Next, a functional model suitable for PPP ambiguity resolution is introduced and a stepwise data processing procedure for PPP ambiguity resolution with the narrow-lane ambiguity fractional bias eliminated is proposed. Finally, after the evaluation of the availability and the reliability for wide-lane ambiguity pseudo-fixing, the spatial correlation of the narrow-lane ambiguity fractional bias is analyzed using a regional network with baseline length between 247 km and 1158 km.

GPS MEASUREMENTS AND HARDWARE BIASES

Un-differenced code and carrier phase measurements from a dual-frequency GPS receiver could be written as:

$$C_1 = \rho + dT - dt + I_1 + \lambda_1 b^r_{C1} - \lambda_1 b^s_{C1} + \varepsilon\ (C_1) \qquad (1)$$

$$P_1 = \rho + dT - dt + I_1 + \lambda_1 b^r_{P1} - \lambda_1 b^s_{P1} + \varepsilon\ (P_1) \qquad (2)$$

$$P_2 = \rho + dT - dt + \frac{f_1^2}{f_2^2} I_1 + \lambda_2 b^r_{P2} - \lambda_2 b^s_{P2} + \varepsilon\ (P_2) \qquad (3)$$

$$\begin{aligned}\Phi_1 = \rho + dT - dt - I_1 + \lambda_1 b^r_{\Phi 1} - \lambda_1 b^s_{\Phi 1} \\ + \lambda_1 N_1 + \varepsilon\ (\Phi_1)\end{aligned} \qquad (4)$$

$$\begin{aligned}\Phi_2 = \rho + dT - dt - \frac{f_1^2}{f_2^2} I_1 + \lambda_2 b^r_{\Phi 2} - \lambda_2 b^s_{\Phi 2} \\ + \lambda_2 N_2 + \varepsilon\ (\Phi_2)\end{aligned} \qquad (5)$$

where

C_i	is the measured C/A-code pseudorange on L_i (m);
P_i	is the measured P-code pseudorange on L_i (m);
ρ	is the true geometric range (m);
dT	is the receiver clock error (m);
dt	is the satellite clock error (m);
I_1	is the ionospheric delay on L_1 (m);
f_i	is the frequency on L_i (Hz);
λ_i	is the wavelength of L_i (m);
N_i	is the integer ambiguity term in measured carrier phase on L_i (cycle);
b^r_*	is the receiver hardware bias on measurement type * (cycle);
b^s_*	is the satellite hardware bias on measurement type * (cycle);
$\varepsilon(.)$	is the noise including residual multipath (m).

The receiver and satellite hardware biases have the following properties:

- They are not necessarily the same for different types of measurement, such as values for $b^r_{C1}, b^r_{P1}, b^r_{P2}, b^r_{\Phi 1}, b^r_{\Phi 2}$ could be different. Similarly, values for $b^s_{C1}, b^s_{P1}, b^s_{P2}, b^s_{\Phi 1}, b^s_{\Phi 2}$ could also be different. All above bias values are not accessible in an absolute sense.
- The receiver hardware bias b^r_* is the same for different satellites measured from a single receiver, which means Satellite-Satellite Single Difference (SSSD) at a single receiver could totally remove this term.
- The satellite hardware bias b^s_* is the same for different receivers, which means Receiver-Receiver Single Difference (RRSD) could totally remove this term.
- After double-difference (between satellite plus between-receiver difference), the receiver and

satellite hardware biases can be completely removed.

HARDWARE BIAS PROPERTIES

In *Wang and Gao* (2007), the test results on the RRSD inter-frequency receiver hardware bias obtained from two different types of receivers in a zero-baseline configuration are investigated. The receivers used are two NovAtel OEM4 receivers and two Javad Legacy receivers configured in the way as shown in Figure 1. The results based on three zero-baseline experiments (Figure 2) indicate that the receiver hardware bias is not stable in response to receiver's power cycle operations and complete signal Loss of Lock (LL). Such hardware bias therefore is not feasible for calibration.

Figure 1: Zero-baseline test configuration to examine receiver hardware bias properties

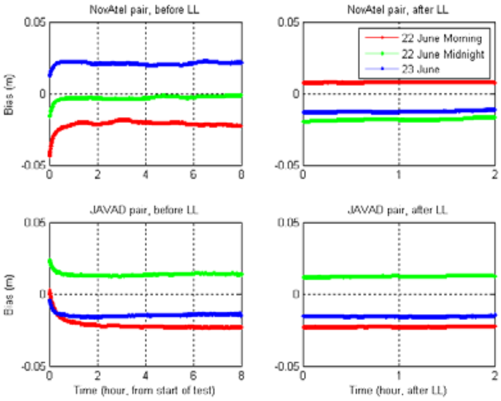

Figure 2: Instability of receiver hardware bias

A total of 46 globally distributed IGS high-rate tracking stations shown in Figure 3 have been used to analyze the stability of the satellite hardware bias. The results indicate that the SSSD WL satellite hardware bias for a particular satellite is spatially stable since the difference between the bias estimates derived from the 46 stations is generally less than 0.2 cycle as seen in Figure 4. The daily stability of SSSD WL satellite hardware bias is fairly good as shown in Figure 5 where the bias variation over the ten days is less than 0.05 cycle. Since the satellite hardware bias is spatially and temporally stable, it can be determined by a reference network and subsequently sent over to PPP users to recover the integer property of the ambiguities.

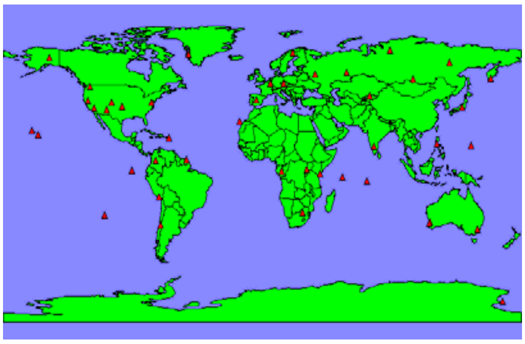

Figure 3: 46 IGS tracking stations to examine satellite hardware bias properties

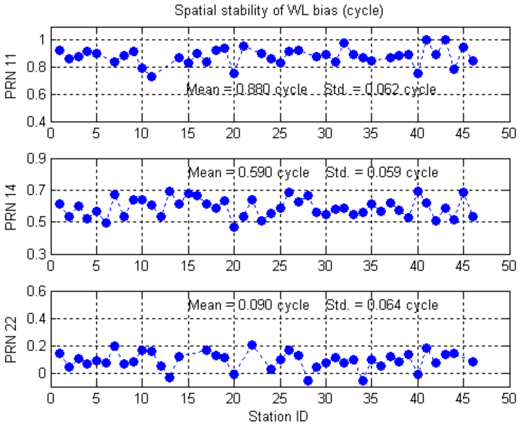

Figure 4: Spatial stability of satellite hardware bias

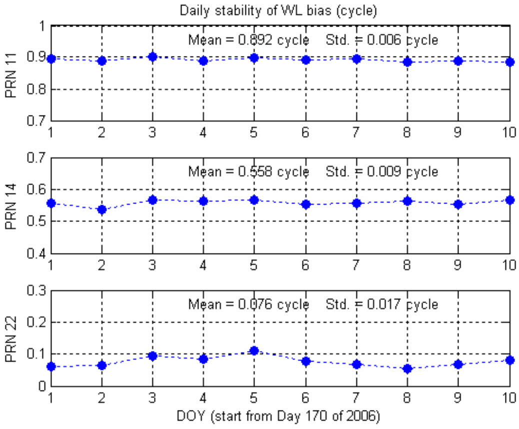

Figure 5: Daily temporal stability of satellite hardware bias

A FUNCTIONAL MODEL FOR PPP AMBIGUITY RESOLUTION

The measurement used in the proposed functional model is the single-difference between satellite ionospheric-free carrier phase combination:

$$\begin{aligned}\nabla\Phi_{IF} &= \frac{f_1^2\nabla\Phi_1 - f_2^2\nabla\Phi_2}{f_1^2 - f_2^2} \\ &= \nabla\rho - \nabla dt + \frac{f_1^2\lambda_1\left(\nabla N_1 - \nabla b_{\Phi 1}^s\right)}{f_1^2 - f_2^2} \\ &\quad - \frac{f_2^2\lambda_2\left(\nabla N_2 - \nabla b_{\Phi 2}^s\right)}{f_1^2 - f_2^2} + \varepsilon\ (\nabla\Phi_{IF}) \\ &= \nabla\rho - \nabla dt + \frac{\gamma\lambda_1 - \lambda_2}{\gamma - 1}\cdot\left(\nabla N_1 - \nabla b_{\Phi_1}^s\right) \\ &\quad + \frac{\lambda_2}{\gamma - 1}\cdot\left(\nabla N_{WL} - \nabla b_{\Phi_{WL}}^s\right) + \varepsilon\ (\nabla\Phi_{IF})\end{aligned} \qquad (6)$$

where $\gamma = \frac{f_1^2}{f_2^2}$. If the ionospheric-free float ambiguity is denoted as

$$\begin{aligned}\nabla A_{IF} &= \frac{\gamma\lambda_1 - \lambda_2}{\gamma - 1}\cdot\left(\nabla N_1 - \nabla b_{\Phi_1}^s\right) \\ &\quad + \frac{\lambda_2}{\gamma - 1}\cdot\left(\nabla N_{WL} - \nabla b_{\Phi_{WL}}^s\right)\end{aligned} \qquad (7)$$

the functional model Eq. (6) can be rewritten as:

$$\nabla\Phi_{IF} = \nabla\rho - \nabla dt + \nabla A_{IF} + \varepsilon\ (\nabla\Phi_{IF}) \qquad (8)$$

In Eq. (7), the coefficient of L1 ambiguity $\left(\nabla N_1 - \nabla b_{\Phi_1}^s\right)$ is equal to $\frac{\gamma\lambda_1 - \lambda_2}{\gamma - 1} = \lambda_{NL} \approx 10.69cm$ and is called narrow-lane ambiguity in the sequel.

Two features can be identified for this functional model:

1) The single differencing process between satellites is applied to eliminate the unstable receiver hardware bias component in the measurement.

2) Only carrier phase measurement is used in this model due to concern that the existence of large multipath in code measurement might break the carrier phase ambiguity constant property if using both code and carrier phase measurements.

A PPP AMBIGUITY RESOLUTION PROCEDURE

Ambiguity resolution procedure based on the proposed functional model includes the following four steps:

1) Ionospheric-free float ambiguity estimation

At the first step, an ionospheric-free float ambiguity ∇A_{IF} is estimated based on the PPP functional model discussed in Eq. (8).

2) Pseudo-fixing the wide-lane ambiguity

At the second step, a pseudo-fixed wide-lane ambiguity is determined using the differential between narrow-lane code and wide-lane carrier phase measurements:

$$\nabla\tilde{N}_{WL} = \nabla N_{WL} + \nabla N_{WL}^{Bias} = Roundoff\left(\frac{\nabla\Phi_{WL} - \nabla P_{NL}}{\lambda_{WL}}\right) \qquad (9)$$

where ∇N_{WL}^{Bias} is the wide-lane integer ambiguity offset introduced by the satellite hardware bias from the narrow-lane code and wide-lane carrier phase measurements

Inserting this pseudo-fixed wide-lane ambiguity $\nabla\tilde{N}_{WL}$ to the expression of ∇A_{IF} leads to:

$$\begin{aligned}\nabla A_{IF} &= \frac{\gamma\lambda_1 - \lambda_2}{\gamma - 1}\cdot\left(\nabla N_1 - \nabla b_{\Phi_1}^s\right) \\ &\quad + \frac{\lambda_2}{\gamma - 1}\cdot\left(\nabla\tilde{N}_{WL} - \nabla N_{WL}^{Bias} - \nabla b_{\Phi_{WL}}^s\right)\end{aligned} \qquad (10)$$

3) Float narrow-lane ambiguity determination

At this step, after replacing the true wide-lane ambiguity component in the ionospheric-free combined ambiguity with the pseudo-fixed one from the second step, a float narrow-lane float ambiguity can be estimated as shown in Eq. (11).

$$\begin{aligned}\nabla A_1 &= \frac{\nabla A_{IF} - \frac{\lambda_2}{\gamma - 1}\cdot\nabla\tilde{N}_{WL}}{\frac{\gamma\lambda_1 - \lambda_2}{\gamma - 1}} \\ &= \frac{\frac{\gamma\lambda_1 - \lambda_2}{\gamma - 1}\cdot\left(\nabla N_1 - \nabla b_{\Phi_1}^s\right) + \frac{\lambda_2}{\gamma - 1}\cdot\left(\nabla\tilde{N}_{WL} - \nabla N_{WL}^{Bias} - \nabla b_{\Phi_{WL}}^s\right)}{\frac{\gamma\lambda_1 - \lambda_2}{\gamma - 1}} \\ &\quad - \frac{\frac{\lambda_2}{\gamma - 1}\cdot\nabla\tilde{N}_{WL}}{\frac{\gamma\lambda_1 - \lambda_2}{\gamma - 1}}\end{aligned}$$

$$= \nabla N_1 - \nabla b^s_{\Phi_1} - \frac{\frac{\lambda_2}{\gamma - 1} \cdot \left(\nabla N^{Bias}_{WL} + \nabla b^s_{\Phi_{WL}} \right)}{\frac{\gamma \lambda_1 - \lambda_2}{\gamma - 1}} \tag{11}$$

Let the last two terms expressed by an integer ∇N^{Bias}_1 and a fractional term $\nabla \delta^{Bias}_1$, Eq. (11) becomes

$$\begin{aligned} \nabla A_1 &= \nabla N_1 + \nabla N^{Bias}_1 + \nabla \delta^{Bias}_1 \\ &= \nabla \tilde{N}_1 + \nabla \delta^{Bias}_1 \end{aligned} \tag{12}$$

The float narrow-lane ambiguity ∇A_1 contains an pseudo integer narrow-lane ambiguity $\nabla \tilde{N}_1$ (we call it a narrow-lane ambiguity because its effective wavelength in the equation is equal to the regular narrow-lane wavelength, and compared to the true narrow-lane ambiguity ∇N_1, it contains an integer narrow-lane ambiguity offset ∇N^{Bias}_1) and an narrow-lane ambiguity fractional bias $\nabla \delta^{Bias}_1$. ∇N^{Bias}_1 and $\nabla \delta^{Bias}_1$ are caused by the wide-lane integer ambiguity offset ∇N^{Bias}_{WL}, satellite hardware bias from the narrow-lane carrier phase $\nabla b^s_{\Phi_1}$ and the satellite hardware bias from the wide-lane carrier phase $\nabla b^s_{\Phi_{WL}}$.

4) Narrow-lane ambiguity resolution

Since the satellite hardware biases ∇N^{Bias}_{WL}, $\nabla b^s_{\Phi_1}$ and $\nabla b^s_{\Phi_{WL}}$ can be considered as constant or stable over time, ∇N^{Bias}_1 and $\nabla \delta^{Bias}_1$ will remain constant or stable over time. If $\nabla \delta^{Bias}_1$ can be determined by a reference network and its value is broadcasted to PPP users for corrections, the integer property of the narrow-lane ambiguity can be recovered as follows:

$$\begin{aligned} \nabla \tilde{N}_1 &= \nabla N_1 + \nabla N^{Bias}_1 \\ &= \nabla A_1 - \nabla \delta^{Bias}_1 \end{aligned} \tag{13}$$

and ambiguity resolution and validation can then be implemented to achieve ambiguity fixed position solutions.

The proposed PPP phase ambiguity resolution method requires a reference network to generate the narrow-lane ambiguity fractional bias corrections which can be obtained as follows:

a) The reference site performs PPP data processing to estimate the narrow-lane ambiguity fractional bias. Since the PPP ambiguity estimates (ionosphere-free ambiguities) at each reference site will become constant values after the position convergence, the float narrow-lane ambiguity can also become constant values as after fixing the wide-lane ambiguity. Therefore the narrow-lane ambiguity fractional bias estimate can be obtained by

$$\nabla \delta^{Bias}_1 = \nabla A_1 - RoundOff\left(\nabla A_1 \right)$$

b) The network data processing center combines the narrow-lane ambiguity fractional bias estimates from all reference sites to generate a network solution for the narrow-lane fractional bias and broadcast it to users as a correction to support PPP phase ambiguity resolution.

PPP WIDE-LANE PSEUDO-FIXING

At the second step of the proposed ambiguity resolution approach, the pseudo-fixing of the wide-lane ambiguity requires application of a correction to the wide-lane ambiguity fractional bias. The wide-lane ambiguity fractional bias for Ashtech Z-XII type receiver on January 6th, 2009 is shown in Figure 6 as an example.

After applying the fractional bias correction, the converged wide-lane float ambiguity will be very close to an integer as shown in Figure 7 and 8. Rounding could be applied in this case for resolve the wide-lane ambiguity (pseudo-fixing). Normally the Time-To-First-Fix (TTFF) the wide-lane ambiguity is below 10 minutes.

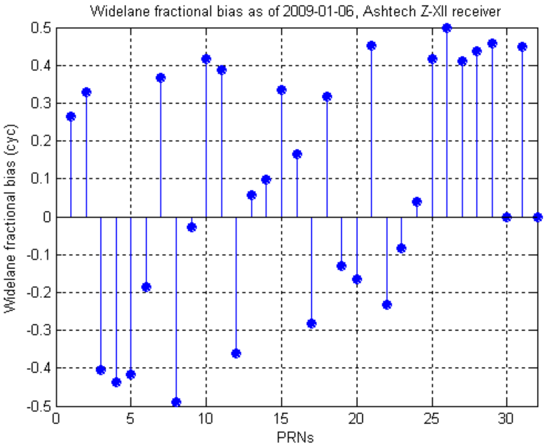

Figure 6: Wide-lane ambiguity fractional bias example

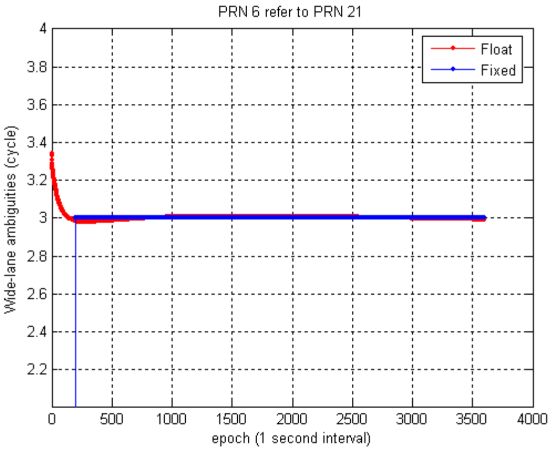

Figure 7: Float versus fixed wide-lane ambiguity for PRN 6

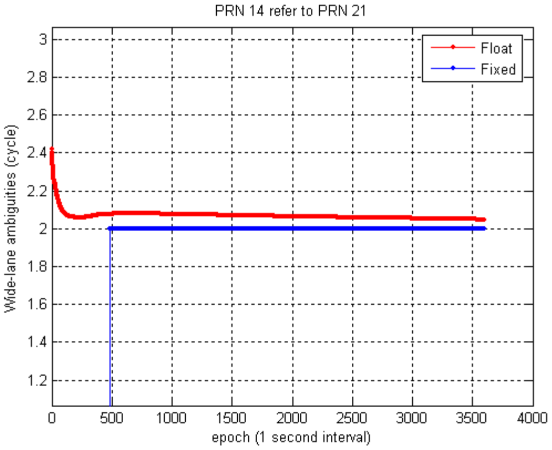

Figure 8: Float versus fixed wide-lane ambiguity for PRN 14

The narrow-lane ambiguity resolution will be conducted only for those ambiguities whose wide-lane component can be pseudo-fixed. Therefore it is necessary to analyze the availability and reliability of the wide-lane ambiguity resolution results. The availability is determined based on the ratio between the total fixed and the total number of ambiguities, accumulated for all epochs over the entire dataset. The reliability is determined based on the ratio between the total correctly fixed and the total fixed number of ambiguities, accumulated for all satellite passes over the entire dataset. Typically the wide-lane ambiguity resolution availability is above 85% and the reliability is close to 100% as shown in Figure 9 based on results for site DHLG as an example.

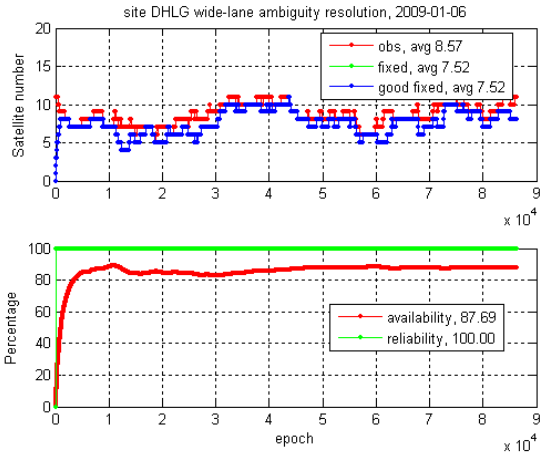

Figure 9: Fixed wide-lane ambiguity availability and reliability at DHLG site

SPATIAL CORRELATION OF PPP NARROW-LANE AMBIGUITY FRACTIONAL BIAS

Some preliminary analysis on narrow-lane ambiguity fractional bias has been conducted based on a regional reference network in North America which consists of 4 IGS high rate sites, namely AMC2, DHLG, GOLD and PIE1. The geographical distribution of these 4 sites is displayed in Figure 10. The minimum baseline length in the reference network is 247 km and the maximum baseline length is 1158 km. From the results based on data acquired on January 6, 2009 and using IGS 15 minutes final orbit and 30 seconds final clock products as shown in Figures 11 to 15, 91.29% of the narrow-lane ambiguity fractional bias estimates from the sites DHLG and GOLD agree better than 0.1 cycle for this 247 km baseline, which demonstrate a high spatial correlation of this fractional bias over the network and the effectiveness of applying this reference-network derived fractional bias by the user to recover the integer property of PPP ambiguities. The spatial correlation is 88.86% between sites AMC2 and PIE1 (baseline length 594 km), 88.58% between sites DHLG and PIE1 (baseline length 716 km), and 89.87% between sites GOLD and PIE1 (baseline length 810 km). Such spatial correlation still remains very high at 86.79% even when the baseline length is extended to 1158 km between sites AMC2 to GOLD

Figure 10: 4 IGS tracking stations to evaluate narrow-lane ambiguity fractional bias spatial correlation

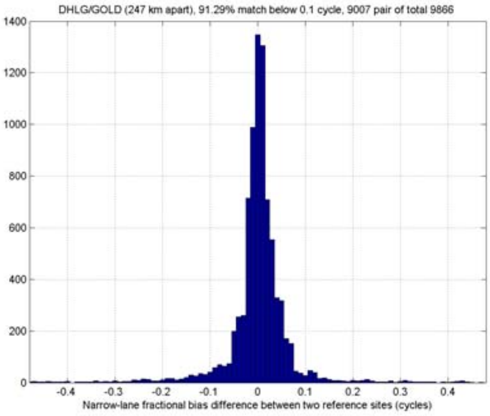

Figure 11: Narrow-lane fractional bias spatial correlation for 247 km baseline formed by DHLG/GOLD

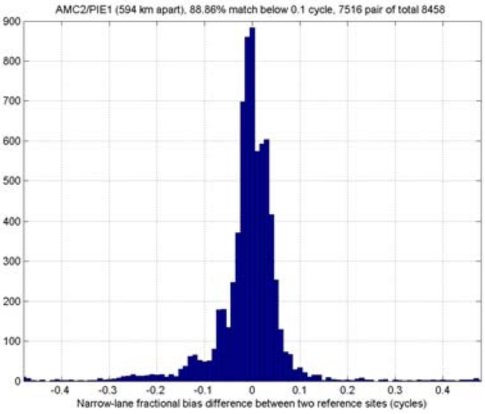

Figure 12: Narrow-lane fractional bias spatial correlation for 594 km baseline formed by AMC2/PIE1

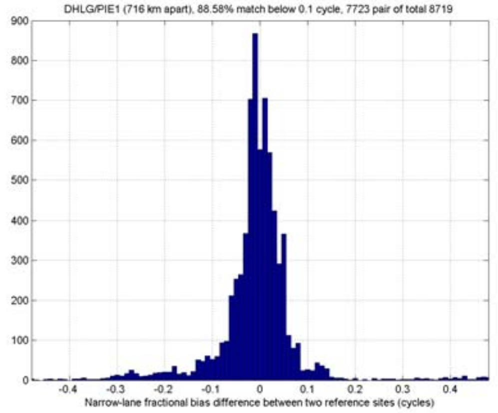

Figure 13: Narrow-lane fractional bias spatial correlation for 716 km baseline formed by DHLG/PIE1

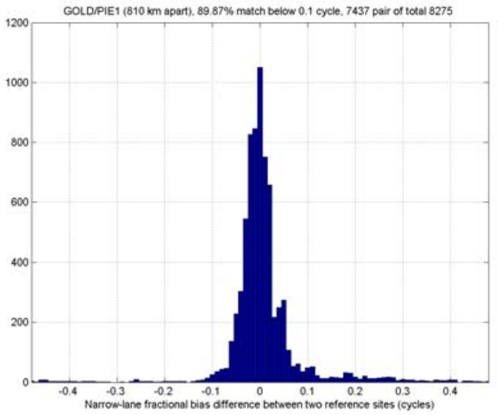

Figure 14: Narrow-lane fractional bias spatial correlation for 810 km baseline formed by GOLD/PIE1

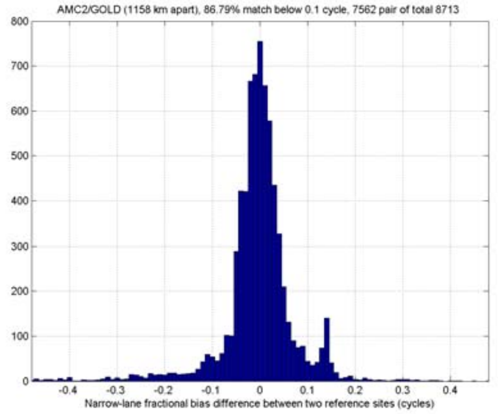

Figure 15: Narrow-lane fractional bias spatial correlation for 1158 km baseline formed by AMC2/GOLD

CONCLUSIONS

A functional model and a stepwise procedure for PPP ambiguity resolution have been proposed in this paper. The results indicate that the wide-lane ambiguity resolution is able to achieve very high availability and reliability, and the spatial correlation of the narrow-lane ambiguity fractional bias remains at a very high level even for baseline of more than 1000 km. The research work to improve PPP float ambiguity quality and adopt the proposed methodologies to generate narrow-lane fractional bias using a reference network and broadcast it to users to recover ambiguity integer property and further to facilitate PPP narrow-lane ambiguity resolution is still on going.

ACKNOWLEDGEMENTS

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